

FUZZY SOFT SETS OVER FULLY UP-SEMIGROUPS



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**A Thesis Submitted to University of Phayao in Partial
Fulfillment of the Requirements for the Master of Science
Degree in Mathematics**

May 2019

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Thesis

Title

Fuzzy Soft Sets over Fully UP-Semigroups

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Master of Science Degree in Mathematics
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ACKNOWLEDGEMENT

First of all, I would like to express my sincere appreciation to my advisor, Assistant Professor Dr. Aiyared Iampan, for his primary idea, guidance and motivation which enable me to carry out my study successfully.

I gladly thank to the supreme committees, Associate Professor Dr. Manoj Siripitukdet, Associate Professor Dr. Tanakit Thianwan, Dr. Teerapong La-Inchua and Assistant Professor Dr. Watcharaporn Cholanjiak, for recommendation about my presentation, report and future works.

I also thank to all of my teachers for their previous valuable lectures that give me more knowledge during my study at the Department of Mathematics, School of Science, University of Phayao.

I am thankful for all my friends with their help and warm friendship. Finally, my graduation would not be achieved without best wish from my parents, who help me everything and always give me greatest love, willpower and financial support until this study completion.

Akarachai Satirad



เรื่อง: เซตอ่อนวิภันนัยเหนือกึ่งกรุปยูทีเต็มรูปแบบ

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คำสำคัญ: พีชคณิตยูที, กึ่งกรุปยูทีเต็มรูปแบบ, เซตวิภันนัย, เซตวิภันนัยลักษณะเฉพาะที่, เซตอ่อนวิภันนัย, แอนด์, ออร์, ยูเนียน(จำกัด), อินเตอร์เซกชัน(ขยาย)

บทคัดย่อ

ในงานวิจัยนี้ เราได้นำเสนอเซตย่อยและเซตวิภันนัยชนิดต่าง ๆ ในกึ่งกรุปยูทีเต็มรูปแบบ เพื่อพิสูจน์ผลลัพธ์ของเซตวิภันนัยภายใต้การดำเนินการอินเตอร์เซกชันและยูเนียน และยังพิจารณาคความสัมพันธ์ระหว่างเซตวิภันนัยลักษณะเฉพาะที่ พีชคณิตย่อยยูทีเอส พีชคณิตย่อยยูทีไอ ตัวกรองยูทีเอสใกล้ ตัวกรองยูทีไอใกล้ ตัวกรองยูทีเอส โอตีสลยูทีเอส โอตีสลยูทีไอ โอตีสลยูทีเอสอย่างเข้ม และโอตีสลยูทีไออย่างเข้ม นอกจากนี้ เรายังได้นำเสนอเซตอ่อนวิภันนัยเหนือกึ่งกรุปยูทีเต็มรูปแบบอีก 10 ชนิด และพิสูจน์สมบัติของเซตอ่อนวิภันนัยภายใต้การดำเนินการอินเตอร์เซกชัน(ขยาย) และยูเนียน(จำกัด) และยังพิจารณาคความสัมพันธ์ระหว่างบางเงื่อนไขของเซตอ่อนวิภันนัย พีชคณิตย่อยยูทีเอสอ่อนวิภันนัย พีชคณิตย่อยยูทีไออ่อนวิภันนัย ตัวกรองยูทีเอสใกล้อ่อนวิภันนัย ตัวกรองยูทีไอใกล้อ่อนวิภันนัย ตัวกรองยูทีเอสอ่อนวิภันนัย ตัวกรองยูทีไออ่อนวิภันนัย โอตีสลยูทีเอสอ่อนวิภันนัย โอตีสลยูทีไออ่อนวิภันนัย โอตีสลยูทีเอสอย่างเข้มอ่อนวิภันนัยและโอตีสลยูทีไออย่างเข้มอ่อนวิภันนัย และสุดท้ายเราได้ใช้กฎการแจกแจงของเซตวิภันนัยใด ๆ เพื่อประยุกต์ไปใช้ในเซตอ่อนวิภันนัยใด ๆ และพิสูจน์สมบัติของบางการดำเนินการสำหรับเซตอ่อนวิภันนัยใด ๆ และศึกษาความเกี่ยวข้องของการดำเนินการต่าง ๆ ได้แก่ ยูเนียน(จำกัด) อินเตอร์เซกชัน(ขยาย) แอนด์ และออร์

Title: FUZZY SOFT SETS OVER FULLY UP–SEMIGROUPS

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Keywords: UP–algebra, fully UP–semigroup, fuzzy set, t –characteristic fuzzy set, fuzzy soft set, AND, OR, (restricted) union, (extended) intersection

ABSTRACT

In this research, we introduce several types of subsets and of fuzzy sets of fully UP–semigroups, investigate the algebraic properties of fuzzy sets under the operations of intersection and union, and discuss the relation between t –characteristic fuzzy sets and UP_s –subalgebras (resp., UP_i –subalgebras, near UP_s –filters, near UP_i –filters, UP_s –filters, UP_i –filters, UP_s –ideals, UP_i –ideals, strongly UP_s –ideals and strongly UP_i –ideals). We introduce ten types of fuzzy soft sets over fully UP–semigroups, investigate the algebraic properties of fuzzy soft sets under the operations of (extended) intersection and (restricted) union, and discuss the relation between some conditions of fuzzy soft sets and fuzzy soft UP_s –subalgebras (resp., fuzzy soft UP_i –subalgebras, fuzzy soft near UP_s –filters, fuzzy soft near UP_i –filters, fuzzy soft UP_s –filters, fuzzy soft UP_i –filters, fuzzy soft UP_s –ideals, fuzzy soft UP_i –ideals, fuzzy soft strongly UP_s –ideals, fuzzy soft strongly UP_i –ideals). We apply distributivity laws of several fuzzy sets for any fuzzy sets and study distributivity laws with any fuzzy soft sets. We investigate properties of some operations for fuzzy soft sets and their interrelation with respect to different operations such as “(restricted) union”, “(extended) intersection”, “AND”, and “OR”.



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CHAPTER I

INTRODUCTION

Among many algebraic structures, algebras of logic form important class of algebras. Examples of these algebras are BCK-algebras [11], BCI-algebras [12], B-algebras [26], UP-algebras [8] and so on. They are strongly connected with some logic. For example, BCI-algebras introduced by Iséki [12] in 1966 have connections with BCI-logic being the BCI-system in combinatory logic which has application in the language of functional programming. BCK and BCI-algebras are two classes of logical algebras. They were introduced by Imai and Iséki [11, 12] in 1966 and have been extensively investigated by many researchers. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras.

Several researches introduced a new class of algebras related to logical algebras and semigroups such as: In 1993, Jun et al. [15] introduced the notion of BCI-semigroups. In 1998, Jun et al. [20] renamed the BCI-semigroup as the IS-algebra. In 2006, Kim [21] introduced the notion of KS-semigroups. In 2015, Endam and Vilela [6] introduced the notion of JB-semigroups. In 2018, Iampan [9] introduced the notion of fully UP-semigroups.

A fuzzy subset F of a set X is a function from X to a closed interval $[0,1]$. The concept of a fuzzy subset of a set was first considered by Zadeh [36] in 1965. The fuzzy set theories developed by Zadeh and others have found many applications in the domain of mathematics and elsewhere. After the introduction of the concept of fuzzy sets by Zadeh [36], several researches were conducted on the generalizations of the notion of fuzzy set and application to many logical algebras such as: In 1998, Jun et al. [14] applied the notion of fuzzy sets to BCI-semigroups (it was renamed as an IS-algebra for the convenience of study), and introduced the concept of fuzzy I-ideals. In 2000, Roh et al. [30] considered

the fuzzification of an associative I-ideal of an IS-algebra. They proved that every fuzzy associative I-ideal is a fuzzy I-ideal. By giving an appropriate example, they verified that a fuzzy I-ideal may not be a fuzzy associative I-ideal. They gave a condition for a fuzzy I-ideal to be a fuzzy associative I-ideal, and they investigated some related properties. In 2003, Jun and Kondo [17] proved that some concepts of BCK/BCI-algebras expressed by a certain formula can be naturally extended to the fuzzy setting and that many results are obtained immediately with the use of our method. Moreover, they proved that these results can be extended to fuzzy IS-algebras. In 2003, Jianming and Dajing [13] introduced the concept of intuitionistic fuzzy associative I-ideals of IS-algebras and they investigated some related properties. In 2007, Prince Williams and Husain [35] studied fuzzy KS-semigroups. In 2016, Endam and Manahon [5] introduced the notion of fuzzy JB-semigroups and they investigated some of its properties.

In 1999, to solve complicated problems in economics, engineering, and environment, we cannot successfully use classical methods because of various uncertainties typical for those problems. Uncertainties cannot be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as the probability theory, the theory of (intuitionistic) fuzzy sets, the theory of vague sets, the theory of interval mathematics, and the theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [25]. In 2001, Maji et al. [24] introduced the concept of fuzzy soft sets as a generalization of the standard soft sets, and presented an application of fuzzy soft sets in a decision making problem. In 2010, Jun et al. [18] applied fuzzy soft set for dealing with several kinds of theories in BCK/BCI-algebras. The notions of fuzzy soft BCK/BCI-algebras, (closed) fuzzy soft ideals and fuzzy soft p-ideals are introduced, and related properties are investigated. In 2013, Rehman et al. [28] studied some operations of fuzzy soft sets and give fundamental properties of fuzzy soft sets. They discuss properties of fuzzy soft sets and

their interrelation with respect to different operations such as union, intersection, restricted union and extended intersection. Then, they illustrate properties of OR, AND operations by giving counter examples. Also we prove that certain De Morgan's laws hold in fuzzy soft set theory with respect to different operations on fuzzy soft sets.



CHAPTER II

REVIEW OF RELATED LITERATURE AND RESEARCH

Two important classes of logical algebras, BCK and BCI-algebras were introduced by Imai and Iséki [11, 12].

Definition 2.0.1 An algebra $A = (A, \cdot, 0)$ is called a *BCI-algebra* if it satisfies the following conditions:

$$\text{(BCI-1)} \quad (\forall x, y, z \in A)((x \cdot y) \cdot (x \cdot z) \cdot (y \cdot z) = 0),$$

$$\text{(BCI-2)} \quad (\forall x, y \in A)((x \cdot (x \cdot y)) \cdot y = x),$$

$$\text{(BCI-3)} \quad (\forall x \in A)(x \cdot x = 0), \text{ and}$$

$$\text{(BCI-4)} \quad (\forall x, y \in A)(x \cdot y = 0, y \cdot x = 0 \Rightarrow x = y).$$

A BCI-algebra A is called a *BCK-algebra* if it satisfies the following identity:

$$\text{(BCK)} \quad (\forall x \in A)(0 \cdot x = 0).$$

In 2002, Neggers and Kim [26] introduced the notion of B-algebras.

Definition 2.0.2 An algebra $A = (A, \cdot, 0)$ of type $(2, 0)$ is called a *B-algebra* if it satisfies the following axioms:

$$\text{(B-1)} \quad (\forall x \in A)(x \cdot x = 0),$$

$$\text{(B-2)} \quad (\forall x \in A)(x \cdot 0 = x), \text{ and}$$

$$\text{(B-3)} \quad (\forall x, y, z \in A)((x \cdot y) \cdot z = x \cdot (z \cdot (0 \cdot y))).$$

In 2017, Iampan [8] introduced the notion of UP-algebras.

Definition 2.0.3 An algebra $A = (A, \cdot, 0)$ of type $(2, 0)$ is called a *UP-algebra* where A is a nonempty set, \cdot is a binary operation on A , and 0 is a fixed element of A (i.e., a nullary operation) if it satisfies the following axioms:

$$\text{(UP-1)} \quad (\forall x, y, z \in A)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0),$$

$$\text{(UP-2)} \quad (\forall x \in A)(0 \cdot x = x),$$

$$\text{(UP-3)} \quad (\forall x \in A)(x \cdot 0 = 0), \text{ and}$$

$$\text{(UP-4)} \quad (\forall x, y \in A)(x \cdot y = 0, y \cdot x = 0 \Rightarrow x = y).$$

In 1993, Jun et al. [15] introduced the notion of BCI-semigroups (it was renamed as IS-algebras for the convenience of study).

Definition 2.0.4 An IS-algebra is a nonempty set A together with two binary operations \cdot and $*$ and a constant 0 satisfying the following:

$$\text{(IS-1)} \quad (A, \cdot, 0) \text{ is a BCI-algebra,}$$

$$\text{(IS-2)} \quad (A, *) \text{ is a semigroup, and}$$

$$\text{(IS-3)} \quad \text{The operation } * \text{ is left and right distributive over the operation } \cdot.$$

In 2006, Kim [21] introduced the notion of KS-semigroups.

Definition 2.0.5 A KS-semigroup is a nonempty set A together with two binary operations \cdot and $*$ and a constant 0 satisfying the following:

$$\text{(KS-1)} \quad (A, \cdot, 0) \text{ is a BCK-algebra,}$$

$$\text{(KS-2)} \quad (A, *) \text{ is a semigroup, and}$$

(KS-3) The operation $*$ is left and right distributive over the operation \cdot .

In 2015, Endam and Vilela [6] introduced the notion of JB-semigroups.

Definition 2.0.6 A JB-semigroup is a nonempty set A together with two binary operations \cdot and $*$ and a constant 0 satisfying the following:

(JB-1) $(A, \cdot, 0)$ is a B-algebra,

(JB-2) $(A, *)$ is a semigroup, and

(JB-3) The operation $*$ is left and right distributive over the operation \cdot .

In 2018, Iampan [9] introduced the notion of fully UP-semigroups (in short, f -UP-semigroups).

Definition 2.0.7 An f -UP-semigroup is a nonempty set A together with two binary operations \cdot and $*$ and a constant 0 satisfying the following:

(fUP-1) $(A, \cdot, 0)$ is a UP-algebra,

(fUP-2) $(A, *)$ is a semigroup, and

(fUP-3) The operation $*$ is left and right distributive over the operation \cdot .

In 1965, Zadeh [36] introduced the concept of a fuzzy set for the first time.

Definition 2.0.8 A *fuzzy set* F in a nonempty set U (or a *fuzzy subset* of U) is described by its membership function f_F . To every point $x \in U$, this function associates a real number $f_F(x)$ in the interval $[0, 1]$. The number $f_F(x)$ is interpreted for the point as a degree of belonging x to the fuzzy set F , that is,

$F := \{(u, f_F(u)) \mid u \in U\}$. If $A \subseteq U$ and $t \in (0, 1]$, the t -characteristic function [19] χ_A^t of U is a function of U into $\{0, t\}$ defined as follows:

$$\chi_A^t(x) = \begin{cases} t & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases}$$

By the definition of t -characteristic function, χ_A^t is a function of U into $\{0, t\} \subset [0, 1]$. We denote the fuzzy set F_A^t in U is described by its membership function χ_A^t , is called the t -characteristic fuzzy set of A in U . We say that a fuzzy set F in U is *constant* if its membership function f_F is constant.

In 1999 - 2004, Jun et al. [29, 16] and Jianming and Dajing [13] applied the notion of fuzzy sets to IS-algebras.

Definition 2.0.9 A fuzzy set F in a semigroup $(A, *)$ is called a *fuzzy stable* if $(\forall x, y \in A)(f_F(x * y) \geq f_F(y))$.

Definition 2.0.10 A fuzzy set F in a BCI-algebra $(A, \cdot, 0)$ is called a *fuzzy sub-algebra* if $(\forall x, y \in A)(f_F(x \cdot y) \geq \min\{f_F(x), f_F(y)\})$.

Definition 2.0.11 A fuzzy set F in a BCI-algebra $(A, \cdot, 0)$ is called a *fuzzy ideal* of A if it satisfies the following conditions:

- (1) $(\forall x \in A)(f_F(0) \geq f_F(x))$, and
- (2) $(\forall x, y \in A)(f_F(x) \geq \min\{f_F(x \cdot y), f_F(y)\})$.

Definition 2.0.12 A fuzzy set F in an IS-algebra $(A, \cdot, *, 0)$ is called a *fuzzy I-ideal* of A if it satisfies the following conditions:

- (1) F is a fuzzy stable, and
- (2) F is a fuzzy ideal of a BCI-algebra A .

Definition 2.0.13 A fuzzy set F in an IS-algebra $(A, \cdot, *, 0)$ is called a *fuzzy associative I-ideal* of A if it satisfies the following conditions:

- (1) F is a fuzzy stable, and
- (2) $(\forall x, y, z \in A)(f_F(x) \geq \min\{f_F((x \cdot y) \cdot z), f_F(y \cdot z)\})$.

In 2016, Endam and Manahon [5] applied the notion of fuzzy sets to JB-semigroups.

Definition 2.0.14 A fuzzy JB-semigroup F of a JB-semigroup $(A, \cdot, *, 0)$ is called a *fuzzy sub JB-semigroup* of A if it satisfies the following conditions:

- (1) $(\forall x, y \in A)(f_F(x \cdot y) \geq \min\{f_F(x), f_F(y)\})$, and
- (2) $(\forall x, y \in A)(f_F(x * y) \geq \min\{f_F(x), f_F(y)\})$.

Definition 2.0.15 A fuzzy JB-semigroup F of a JB-semigroup $(A, \cdot, *, 0)$ is called a *fuzzy JB-ideal* of A if it satisfies the following conditions:

- (1) $(\forall x, y, a, b \in A)(f_F((x \cdot a) \cdot (y \cdot b)) \geq \min\{f_F(x \cdot y), f_F(a \cdot b)\})$, and
- (2) $(\forall x, y \in A)(f_F(x * y) \geq \min\{f_F(x), f_F(y)\})$.

Definition 2.0.16 A fuzzy JB-semigroup F of a JB-semigroup $(A, \cdot, *, 0)$ is called a *fuzzy JB_s -ideal* of A if it satisfies the following conditions:

- (1) $(\forall x, y, a, b \in A)(f_F((x \cdot a) \cdot (y \cdot b)) \geq \min\{f_F(x \cdot y), f_F(a \cdot b)\})$, and
- (2) $(\forall x, y \in A)(f_F(x * y) \geq \max\{f_F(x), f_F(y)\})$.

In 2001, Maji et al. [24] introduced the notion of fuzzy soft sets, as a generalization of the standard soft sets.

Definition 2.0.17 Let U be an initial universe set and P be a set of parameters. Let $F(U)$ denote the set of all fuzzy sets in U . Then (\tilde{F}, E) is called a *fuzzy soft set* over U where $E \subseteq P$ and \tilde{F} is a mapping given by $\tilde{F}: E \rightarrow F(U)$.

In general, for every $e \in E$, a fuzzy set,

$$\tilde{F}[e] := \{(u, f_{\tilde{F}[e]}(u)) \mid u \in U\}$$

in U is called *fuzzy value set* of parameter e .

In 2010, Jun et al. [18] applied the notion of fuzzy soft sets to BCK/BCI-algebras.

Definition 2.0.18 Let (\tilde{F}, E) be a fuzzy soft set over a BCK/BCI-algebra $(A, \cdot, 0)$ where E is a subset of P . If there exists $e \in E$ such that $\tilde{F}[e]$ is a fuzzy BCK/BCI-algebra in A , we say that (\tilde{F}, E) is a fuzzy soft BCK/BCI-algebra based on a parameter e over A . If (\tilde{F}, E) is a fuzzy soft BCK/BCI-algebra based on a parameter e over A for all $e \in E$, we say that (\tilde{F}, E) is a fuzzy soft BCK/BCI-algebra over A .

Definition 2.0.19 Let (\tilde{F}, E) be a fuzzy soft set over a BCK/BCI-algebra $(A, \cdot, 0)$ where E is a subset of P . If there exists $e \in E$ such that $\tilde{F}[e]$ is a fuzzy ideal of A , we say that (\tilde{F}, E) is a fuzzy soft ideal of A based on a parameter e . If (\tilde{F}, E) is a fuzzy soft ideal of A based on all parameters, we say that (\tilde{F}, E) is a fuzzy soft ideal of A .

CHAPTER III

PRELIMINARIES

Before we begin our study, we will introduce a UP-algebra. From [8], we know that the notion of UP-algebras is a generalization of KU-algebras (see [27]).

On a UP-algebra $A = (A, \cdot, 0)$, we define a binary relation \leq on A as follows:

$$(\forall x, y \in A)(x \leq y \Leftrightarrow x \cdot y = 0).$$

Example 3.0.20 [33] Let X be a universal set and let $\Omega \in \mathcal{P}(X)$. Let $\mathcal{P}_\Omega(X) = \{A \in \mathcal{P}(X) \mid \Omega \subseteq A\}$. Define a binary operation \cdot on $\mathcal{P}_\Omega(X)$ by putting $A \cdot B = B \cap (A' \cup \Omega)$ for all $A, B \in \mathcal{P}_\Omega(X)$. Then $(\mathcal{P}_\Omega(X), \cdot, \Omega)$ is a UP-algebra and we shall call it the *generalized power UP-algebra of type 1 with respect to Ω* .

Example 3.0.21 [33] Let X be a universal set and let $\Omega \in \mathcal{P}(X)$. Let $\mathcal{P}^\Omega(X) = \{A \in \mathcal{P}(X) \mid A \subseteq \Omega\}$. Define a binary operation $*$ on $\mathcal{P}^\Omega(X)$ by putting $A * B = B \cup (A' \cap \Omega)$ for all $A, B \in \mathcal{P}^\Omega(X)$. Then $(\mathcal{P}^\Omega(X), *, \Omega)$ is a UP-algebra and we shall call it the *generalized power UP-algebra of type 2 with respect to Ω* .

In particular, we have $(\mathcal{P}(X), \cdot, \emptyset)$ is the power UP-algebra of type 1 and $(\mathcal{P}(X), *, X)$ is the power UP-algebra of type 2.

Example 3.0.22 [4] Let \mathbb{N} be the set of all natural numbers with two binary operations \circ and \bullet defined by,

$$(\forall x, y \in \mathbb{N}) \left(x \circ y = \begin{cases} y & \text{if } x < y, \\ 0 & \text{otherwise} \end{cases} \right),$$

and

$$(\forall x, y \in \mathbb{N}) \left(x \bullet y = \begin{cases} y & \text{if } x > y \text{ or } x = 0, \\ 0 & \text{otherwise} \end{cases} \right).$$

Then $(\mathbb{N}, \circ, 0)$ and $(\mathbb{N}, \bullet, 0)$ are UP-algebras.

For more examples of UP-algebras, see [3, 9, 32, 33].

In a UP-algebra $A = (A, \cdot, 0)$, the following assertions are valid (see [8, 9]).

$$(\forall x \in A)(x \cdot x = 0), \tag{3.0.1}$$

$$(\forall x, y, z \in A)(x \cdot y = 0, y \cdot z = 0 \Rightarrow x \cdot z = 0), \tag{3.0.2}$$

$$(\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow (z \cdot x) \cdot (z \cdot y) = 0), \tag{3.0.3}$$

$$(\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow (y \cdot z) \cdot (x \cdot z) = 0), \tag{3.0.4}$$

$$(\forall x, y \in A)(x \cdot (y \cdot x) = 0), \tag{3.0.5}$$

$$(\forall x, y \in A)((y \cdot x) \cdot x = 0 \Leftrightarrow x = y \cdot x), \tag{3.0.6}$$

$$(\forall x, y \in A)(x \cdot (y \cdot y) = 0), \tag{3.0.7}$$

$$(\forall a, x, y, z \in A)((x \cdot (y \cdot z)) \cdot (x \cdot ((a \cdot y) \cdot (a \cdot z))) = 0), \tag{3.0.8}$$

$$(\forall a, x, y, z \in A)((((a \cdot x) \cdot (a \cdot y)) \cdot z) \cdot ((x \cdot y) \cdot z) = 0), \tag{3.0.9}$$

$$(\forall x, y, z \in A)((x \cdot y) \cdot z \cdot (y \cdot z) = 0), \tag{3.0.10}$$

$$(\forall x, y, z \in A)(x \cdot y = 0 \Rightarrow x \cdot (z \cdot y) = 0), \tag{3.0.11}$$

$$(\forall x, y, z \in A)((x \cdot y) \cdot z \cdot (x \cdot (y \cdot z)) = 0), \text{ and} \tag{3.0.12}$$

$$(\forall a, x, y, z \in A)((x \cdot y) \cdot z \cdot (y \cdot (a \cdot z)) = 0). \tag{3.0.13}$$

Definition 3.0.23 [8, 34, 7, 10] A nonempty subset S of a UP-algebra $(A, \cdot, 0)$ is called

- (1) a *UP-subalgebra* of A if $(\forall x, y \in S)(x \cdot y \in S)$,
- (2) a *near UP-filter* of A if it satisfies the following properties:
 - (i) the constant 0 of A is in S , and
 - (ii) $(\forall x, y \in A)(y \in S \Rightarrow x \cdot y \in S)$,
- (3) a *UP-filter* of A if it satisfies the following properties:
 - (i) the constant 0 of A is in S , and
 - (ii) $(\forall x, y \in A)(x \cdot y \in S, x \in S \Rightarrow y \in S)$,
- (4) a *UP-ideal* of A if it satisfies the following properties:
 - (i) the constant 0 of A is in S , and
 - (ii) $(\forall x, y, z \in A)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S)$,
- (5) a *strongly UP-ideal* of A if it satisfies the following properties:
 - (i) the constant 0 of A is in S , and
 - (ii) $(\forall x, y, z \in A)((z \cdot y) \cdot (z \cdot x) \in S, y \in S \Rightarrow x \in S)$.

We know that the notion of UP-subalgebras is a generalization of near UP-filters, the notion of near UP-filters is a generalization of UP-filters, the notion of UP-filters is a generalization of UP-ideals, and the notion of UP-ideals is a generalization of strongly UP-ideals. Moreover, they also proved that a UP-algebra A is the only one strongly UP-ideal of itself.

Definition 3.0.24 A nonempty subset S of a semigroup $(A, *)$ is called

- (1) a *subsemigroup* of A if $(\forall x, y \in S)(x * y \in S)$, and
- (2) an *ideal* of A if $(\forall x \in A, \forall s \in S)(x * s, s * x \in S)$.

Clearly, an ideal is a subsemigroup.

Lemma 3.0.25 *Let S be a nonempty subset of a UP-algebra $(A, \cdot, 0)$ and $t \in (0, 1]$. Then the constant 0 of A is in S if and only if $(\forall x \in A)(\chi_S^t(0) \geq \chi_S^t(x))$.*

Proof. Assume that $0 \in S$. Then for all $x \in A$, $\chi_S^t(0) = t \geq \chi_S^t(x)$.

Conversely, assume that $\chi_S^t(0) \geq \chi_S^t(x)$ for all $x \in A$. Since S is a nonempty subset of A , we have an element a in S , that is, $\chi_S^t(a) = t$. Thus $t \geq \chi_S^t(0) \geq \chi_S^t(a) = t$. So $\chi_S^t(0) = t$, that is, $0 \in S$. \square

Definition 3.0.26 ([34, 7]) A fuzzy set F in a UP-algebra $A = (A, \cdot, 0)$ is called

- (1) a *fuzzy UP-subalgebra* of A if $(\forall x, y \in A)(f_F(x \cdot y) \geq \min\{f_F(x), f_F(y)\})$,
- (2) a *fuzzy UP-filter* of A if
 - (i) $(\forall x \in A)(f_F(0) \geq f_F(x))$, and
 - (ii) $(\forall x, y \in A)(f_F(y) \geq \min\{f_F(x \cdot y), f_F(x)\})$,
- (3) a *fuzzy UP-ideal* of A if
 - (i) $(\forall x \in A)(f_F(0) \geq f_F(x))$, and
 - (ii) $(\forall x, y, z \in A)(f_F(x \cdot z) \geq \min\{f_F(x \cdot (y \cdot z)), f_F(y)\})$,
- (4) a *fuzzy strongly UP-ideal* of A if
 - (i) $(\forall x \in A)f_F(0) \geq f_F(x)$, and
 - (ii) $(\forall x, y, z \in A)(f_F(x) \geq \min\{f_F((z \cdot y) \cdot (z \cdot x)), f_F(y)\})$.

Now, we introduce the notion of fuzzy near UP-filters of UP-algebras as follows:

Definition 3.0.27 A fuzzy set F in a UP-algebra $A = (A, \cdot, 0)$ is called a *fuzzy near UP-filter* of A if

- (i) $(\forall x \in A)(f_F(0) \geq f_F(x))$, and
- (ii) $(\forall x, y \in A)(f_F(x \cdot y) \geq f_F(y))$.

We know that the notion of fuzzy UP-subalgebras is a generalization of fuzzy near UP-filters, the notion of fuzzy near UP-filters is a generalization of fuzzy UP-filters, the notion of fuzzy UP-filters is a generalization of fuzzy UP-ideals, and the notion of fuzzy UP-ideals is a generalization of fuzzy strongly UP-ideals. Moreover, fuzzy strongly UP-ideals and constant fuzzy sets coincide in UP-algebras.

Theorem 3.0.28 [7] *Fuzzy strongly UP-ideals and constant fuzzy sets coincide in UP-algebras.*

Theorem 3.0.29 *Let S be a nonempty subset of a UP-algebra $A = (A, \cdot, 0)$ and $t \in (0, 1]$. Then the following statements hold:*

- (1) *S is a UP-subalgebra of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy UP-subalgebra of A ,*
- (2) *S is a near UP-filter of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy near UP-filter of A ,*
- (3) *S is a UP-filter of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy UP-filter of A ,*
- (4) *S is a UP-ideal of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy UP-ideal of A , and*
- (5) *S is a strongly UP-ideal of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy strongly UP-ideal of A .*

Proof. (1) Assume that S is a UP-subalgebra of A . Let $x, y \in A$.

Case 1: $x, y \in S$. Then $\chi_S^t(x) = t = \chi_S^t(y)$, so $\min\{\chi_S^t(x), \chi_S^t(y)\} = t$. Since S is a UP-subalgebra of A , we have $x \cdot y \in S$ and so $\chi_S^t(x \cdot y) = t$. Therefore, $\chi_S^t(x \cdot y) = t \geq t = \min\{\chi_S^t(x), \chi_S^t(y)\}$.

Case 2: $x \notin S$ or $y \notin S$. Then $\chi_S^t(x) = 0$ or $\chi_S^t(y) = 0$, so

$$\min\{\chi_S^t(x), \chi_S^t(y)\} = 0.$$

Therefore, $\chi_S^t(x \cdot y) \geq 0 = \min\{\chi_S^t(x), \chi_S^t(y)\}$.

Hence, F_S^t is a fuzzy UP-subalgebra of A .

Conversely, assume that F_S^t is a fuzzy UP-subalgebra of A . Let $x, y \in S$. Then $\chi_S^t(y) = t = \chi_S^t(x)$, so $\min\{\chi_S^t(x), \chi_S^t(y)\} = t$. Since F_S^t is a fuzzy UP-subalgebra of A , we have $t \geq \chi_S^t(x \cdot y) \geq \min\{\chi_S^t(x), \chi_S^t(y)\} = t$. Thus $\chi_S^t(x \cdot y) = t$, that is, $x \cdot y \in S$. Hence, S is a UP-subalgebra of A .

(2) Assume that S is a near UP-filter of A . Since $0 \in S$, it follows from Lemma 3.0.25 that $\chi_S^t(0) \geq \chi_S^t(x)$ for all $x \in A$. Next, let $x, y \in A$.

Case 1: $y \in S$. Then $\chi_S^t(y) = t$. Since S is a near UP-filter of A , we have $x \cdot y \in S$ and so $\chi_S^t(x \cdot y) = t$. Therefore, $\chi_S^t(x \cdot y) = t \geq t = \chi_S^t(y)$.

Case 2: $y \notin S$. Then $\chi_S^t(y) = 0$. Thus $\chi_S^t(x \cdot y) \geq 0 = \chi_S^t(y)$.

Hence, F_S^t is a fuzzy near UP-filter of A .

Conversely, assume that F_S^t is a fuzzy near UP-filter of A . Since $\chi_S^t(0) \geq \chi_S^t(x)$ for all $x \in A$, it follows from Lemma 3.0.25 that $0 \in S$. Next, let $x, y \in A$ be such that $y \in S$. Then $\chi_S^t(y) = t$. Since F_S^t is a fuzzy near UP-filter of A , we have $t \geq \chi_S^t(x \cdot y) \geq \chi_S^t(y) = t$. Thus $\chi_S^t(x \cdot y) = t$, that is, $x \cdot y \in S$. Hence, S is a near UP-filter of A .

(3) Assume that S is a UP-filter of A . Since $0 \in S$, it follows from Lemma

3.0.25 that $\chi_S^t(0) \geq \chi_S^t(x)$ for all $x \in A$. Next, let $x, y \in A$.

Case 1: $x, y \in S$. Then $\chi_S^t(x) = t = \chi_S^t(y)$. Thus $\chi_S^t(y) = t \geq \chi_S^t(x \cdot y) = \min\{\chi_S^t(x \cdot y), \chi_S^t(x)\}$.

Case 2: $x \notin S$ or $y \notin S$. If $x \notin S$, then $\chi_S^t(x) = 0$. Thus $\chi_S^t(y) \geq 0 = \min\{\chi_S^t(x \cdot y), \chi_S^t(x)\}$. If $y \notin S$, then $\chi_S^t(y) = 0$. Since S is a UP-filter of A , we have $x \cdot y \notin S$ or $x \notin S$ and so $\chi_S^t(x \cdot y) = 0$ or $\chi_S^t(x) = 0$. Thus $\chi_S^t(y) = 0 \geq 0 = \min\{\chi_S^t(x \cdot y), \chi_S^t(x)\}$.

Hence, F_S^t is a fuzzy UP-filter of A .

Conversely, assume that F_S^t is a fuzzy UP-filter of A . Since $\chi_S^t(0) \geq \chi_S^t(x)$ for all $x \in A$, it follows from Lemma 3.0.25 that $0 \in S$. Next, let $x, y \in A$ be such that $x \cdot y \in S$ and $x \in S$. Then $\chi_S^t(x \cdot y) = t = \chi_S^t(x)$, so $\min\{\chi_S^t(x \cdot y), \chi_S^t(x)\} = t$. Since F_S^t is a fuzzy UP-filter of A , we have $t \geq \chi_S^t(y) \geq \min\{\chi_S^t(x \cdot y), \chi_S^t(x)\} = t$. Thus $\chi_S^t(y) = t$, that is, $y \in S$. Hence, S is a UP-filter of A .

(4) Assume that S is a UP-ideal of A . Since $0 \in S$, it follows from Lemma 3.0.25 that $\chi_S^t(0) \geq \chi_S^t(x)$ for all $x \in A$. Next, let $x, y, z \in A$.

Case 1: $x \cdot (y \cdot z), y \in S$. Then $\chi_S^t(x \cdot (y \cdot z)) = t = \chi_S^t(y)$, so $\min\{\chi_S^t(x \cdot (y \cdot z)), \chi_S^t(y)\} = t$. Since S is a UP-ideal of A , we have $x \cdot z \in S$ and so $\chi_S^t(x \cdot z) = t$. Thus $\chi_S^t(x \cdot z) = t \geq t = \min\{\chi_S^t(x \cdot (y \cdot z)), \chi_S^t(y)\}$.

Case 2: $x \cdot (y \cdot z) \notin S$ or $y \notin S$. Then $\chi_S^t(x \cdot (y \cdot z)) = 0$ or $\chi_S^t(y) = 0$, so $\min\{\chi_S^t(x \cdot (y \cdot z)), \chi_S^t(y)\} = 0$. Thus $\chi_S^t(x \cdot z) \geq 0 = \min\{\chi_S^t(x \cdot (y \cdot z)), \chi_S^t(y)\}$.

Hence, F_S^t is a fuzzy UP-ideal of A .

Conversely, assume that F_S^t is a fuzzy UP-ideal of A . Since $\chi_S^t(0) \geq \chi_S^t(x)$ for all $x \in A$, it follows from Lemma 3.0.25 that $0 \in S$. Next, let $x, y, z \in A$ such that $x \cdot (y \cdot z) \in S$ and $y \in S$. Then $\chi_S^t(x \cdot (y \cdot z)) = t = \chi_S^t(y)$, so

$\min\{\chi_S^t(x \cdot (y \cdot z)), \chi_S^t(y)\} = t$. Since F_S^t is a fuzzy UP-ideal of A , we have $t \geq \chi_S^t(x \cdot z) \geq \min\{\chi_S^t(x \cdot (y \cdot z)), \chi_S^t(y)\} = t$. Thus $\chi_S^t(x \cdot z) = t$, that is, $x \cdot z \in S$. Hence, S is a UP-ideal of A .

(5) It is straightforward by Theorem 3.0.28, and A is the only one strongly UP-ideal of itself. \square

Definition 3.0.30 [31] A fuzzy set F in a semigroup $A = (A, *)$ is called

(1) a *fuzzy subsemigroup* of A if

$$(\forall x, y \in A)(f_F(x * y) \geq \min\{f_F(x), f_F(y)\}), \text{ and}$$

(2) a *fuzzy ideal* of A if for any $x, y \in A$,

$$(\forall x, y \in A)(f_F(x * y) \geq \max\{f_F(x), f_F(y)\}).$$

Clearly, a fuzzy ideal is a fuzzy subsemigroup.

Theorem 3.0.31 Let S be a nonempty subset of a semigroup $A = (A, *)$ and $t \in (0, 1]$. Then the following statements hold:

(1) S is a subsemigroup of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy subsemigroup of A , and

(2) S is an ideal of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy ideal of A .

Proof. (1) Assume that S is a subsemigroup of A . Let $x, y \in A$.

Case 1: $x, y \in S$. Then $\chi_S^t(x) = t = \chi_S^t(y)$, so $\min\{\chi_S^t(x), \chi_S^t(y)\} = t$. Since S is a subsemigroup of A , we have $x * y \in S$ and so $\chi_S^t(x * y) = t$. Therefore, $\chi_S^t(x * y) = t \geq t = \min\{\chi_S^t(x), \chi_S^t(y)\}$.

Case 2: $x \notin S$ or $y \notin S$. Then $\chi_S^t(x) = 0$ or $\chi_S^t(y) = 0$, so

$$\min\{\chi_S^t(x), \chi_S^t(y)\} = 0.$$

Therefore, $\chi_S^t(x * y) \geq 0 = \min\{\chi_S^t(x), \chi_S^t(y)\}$.

Hence, F_S^t is a fuzzy subsemigroup of A .

Conversely, assume that F_S^t is a fuzzy subsemigroup of A . Let $x, y \in S$. Then $\chi_S^t(y) = t = \chi_S^t(x)$, so $\min\{\chi_S^t(x), \chi_S^t(y)\} = t$. Since F_S^t is a fuzzy subsemigroup of A , we have $t \geq \chi_S^t(x * y) \geq \min\{\chi_S^t(x), \chi_S^t(y)\} = t$. Thus $\chi_S^t(x * y) = t$, that is, $x * y \in S$. Hence, S is a subsemigroup of A .

(2) Assume that S is an ideal of A . Let $x, y \in A$.

Case 1: $x, y \in S$. Then $\chi_S^t(x) = t = \chi_S^t(y)$, so $\max\{\chi_S^t(x), \chi_S^t(y)\} = t$. Since S is an ideal of A , we have $x * y \in S$ and so $\chi_S^t(x * y) = t$. Therefore, $\chi_S^t(x * y) = t \geq t = \max\{\chi_S^t(x), \chi_S^t(y)\}$.

Case 2: $x \notin S$ or $y \notin S$. If $x * y \in S$, then $\chi_S^t(x * y) = t$. Therefore, $\chi_S^t(x * y) = t \geq \max\{\chi_S^t(x), \chi_S^t(y)\}$. If $x * y \notin S$, then $x, y \notin S$. Thus $\chi_S^t(x * y) = 0$ and $\chi_S^t(x) = 0 = \chi_S^t(y)$. Therefore, $\chi_S^t(x * y) = 0 \geq 0 = \max\{\chi_S^t(x), \chi_S^t(y)\}$.

Hence, F_S^t is a fuzzy ideal of A .

Conversely, assume that F_S^t is a fuzzy ideal of A . Let $s \in S$ and $x \in A$. Then $\chi_S^t(s) = t$, so $\max\{\chi_S^t(s), \chi_S^t(x)\} = t$. Since F_S^t is a fuzzy ideal of A , we have $t \geq \chi_S^t(s * x), \chi_S^t(x * s) \geq \max\{\chi_S^t(s), \chi_S^t(x)\} = t$. Thus $\chi_S^t(s * x) = t = \chi_S^t(x * s)$, that is $s * x, x * s \in S$. Hence, S is an ideal of A . \square

Definition 3.0.32 [23] Let $\{F_i\}_{i \in I}$ be a nonempty family of fuzzy sets in a nonempty set U where I is an arbitrary index set. The *intersection* of F_i , denoted by $\bigcap_{i \in I} F_i$, is described by its membership function $f_{\bigcap_{i \in I} F_i}$ which defined

as follows:

$$(\forall x \in U)(f_{\bigcap_{i \in I} F_i}(x) = \inf\{f_{F_i}(x)\}_{i \in I}).$$

The *union* of F_i , denoted by $\bigcup_{i \in I} F_i$, is described by its membership function $f_{\bigcup_{i \in I} F_i}$ which defined as follows:

$$(\forall x \in U)(f_{\bigcup_{i \in I} F_i}(x) = \sup\{f_{F_i}(x)\}_{i \in I}).$$

Definition 3.0.33 [23] Let F and G be fuzzy sets in a nonempty set U . Then $F \leq G$ is defined by $f_F(x) \leq f_G(x)$ for all $x \in U$.

Definition 3.0.34 [22] Let F and G be fuzzy sets in a semigroup $A = (A, *)$. Then the *product* of F and G , denoted by $F \circ G$, is described by their membership function f_F and f_G , respectively which defined as follows: For all $x \in A$,

$$(f_F \circ f_G)(x) = \begin{cases} \sup\{\min\{f_F(y), f_G(z)\}\}_{x=y*z} & \text{if } \exists y, z \in A \text{ such that } x = y * z, \\ 0 & \text{otherwise.} \end{cases}$$

Definition 3.0.35 [22] The semigroup A itself is a fuzzy set of A , denoted by A such that $f_A(x) = 1$ for all $x \in A$.

Lemma 3.0.36 [22] Let F be a fuzzy set in a semigroup $A = (A, *)$. Then

(1) F is a fuzzy subsemigroup of A if and only if it satisfies the condition

$$F \circ F \leq F. \tag{3.0.14}$$

(2) F is a fuzzy ideal of A if and only if it satisfies the condition

$$A \circ F \leq F \text{ and } F \circ A \leq F. \tag{3.0.15}$$

Theorem 3.0.37 Let F_i and F be fuzzy sets in a nonempty set X where I is a nonempty set. Then the following properties hold:

$$(1) F \cap (\bigcup_{i \in I} F_i) = \bigcup_{i \in I} (F \cap F_i),$$

$$(2) (\bigcup_{i \in I} F_i) \cap F = \bigcup_{i \in I} (F_i \cap F),$$

$$(3) F \cup (\bigcap_{i \in I} F_i) = \bigcap_{i \in I} (F \cup F_i), \text{ and}$$

$$(4) (\bigcap_{i \in I} F_i) \cup F = \bigcap_{i \in I} (F_i \cup F).$$

Proof. Let $x \in X$. (1) First, we investigate left hand side of the equality. Assume that $\bigcup_{i \in I} F_i = F^\cup$. Then $F \cap (\bigcup_{i \in I} F_i) = F \cap F^\cup$. Also,

$$\begin{aligned} f_{F \cap F^\cup}(x) &= \min\{f_{F(x)}, f_{F^\cup}(x)\} \\ &= \min\{f_{F(x)}, f_{\bigcup_{i \in I} F_i}(x)\} \\ &= \min\{f_{F(x)}, \sup\{f_{F_i}(x)\}_{i \in I}\}. \end{aligned}$$

Consider the right hand side of the equality. Assume that $F \cap F_i = F_i^\cap$ for all $i \in I$. Then

$$\begin{aligned} f_{\bigcup_{i \in I} F_i^\cap}(x) &= \sup\{f_{F_i^\cap}(x)\}_{i \in I} \\ &= \sup\{f_{F \cap F_i}(x)\}_{i \in I} \\ &= \sup\{\min\{f_F(x), f_{F_i}(x)\}\}_{i \in I}. \end{aligned}$$

It is clear that $\min\{f_{F(x)}, \sup\{f_{F_i}(x)\}_{i \in I}\} = \sup\{\min\{f_F(x), f_{F_i}(x)\}\}_{i \in I}$. Therefore, $F \cap (\bigcup_{i \in I} F_i) = \bigcup_{i \in I} (F \cap F_i)$.

(2) By using techniques as in (1), then (2) can be derived.

(3) First, we investigate left hand side of the equality. Assume that $\bigcap_{i \in I} F_i = F^\cap$. Then $F \cup (\bigcap_{i \in I} F_i) = F \cup F^\cap$. Also,

$$\begin{aligned} f_{F \cup F^\cap}(x) &= \max\{f_{F(x)}, f_{F^\cap}(x)\} \\ &= \max\{f_{F(x)}, f_{\bigcap_{i \in I} F_i}(x)\} \end{aligned}$$

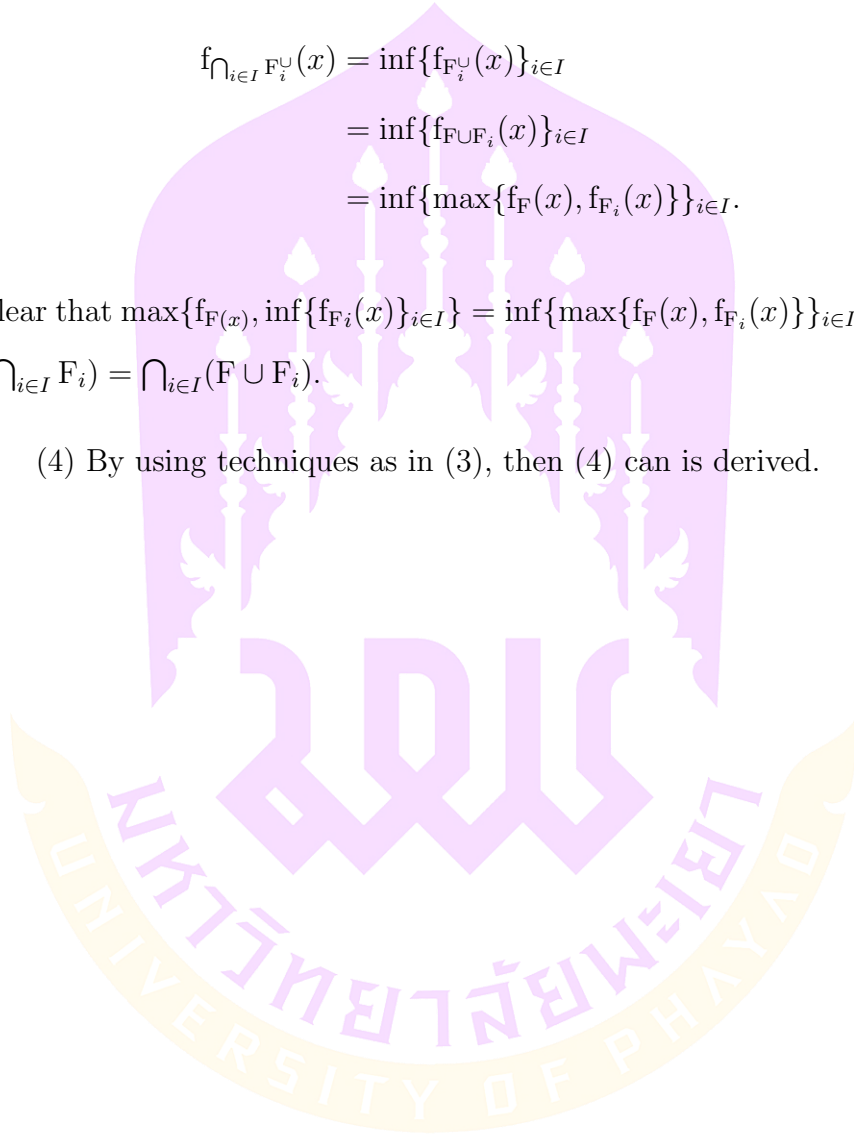
$$= \max\{f_{F(x)}, \inf\{f_{F_i(x)}\}_{i \in I}\}.$$

Consider the right hand side of the equality. Assume that $F \cup F_i = F_i^\cup$ for all $i \in I$. Then

$$\begin{aligned} f_{\bigcap_{i \in I} F_i^\cup}(x) &= \inf\{f_{F_i^\cup}(x)\}_{i \in I} \\ &= \inf\{f_{F \cup F_i}(x)\}_{i \in I} \\ &= \inf\{\max\{f_F(x), f_{F_i}(x)\}\}_{i \in I}. \end{aligned}$$

It is clear that $\max\{f_{F(x)}, \inf\{f_{F_i}(x)\}_{i \in I}\} = \inf\{\max\{f_F(x), f_{F_i}(x)\}\}_{i \in I}$. Therefore, $F \cup (\bigcap_{i \in I} F_i) = \bigcap_{i \in I} (F \cup F_i)$.

(4) By using techniques as in (3), then (4) can be derived. \square



CHAPTER IV

RESULTS

4.1 Special subsets of fully UP-semigroups

In this section, we introduce the notions of UP_s -subalgebras, UP_i -subalgebras, near UP_s -filters, near UP_i -filters, UP_s -filters, UP_i -filters, UP_s -ideals, UP_i -ideals, strongly UP_s -ideals, and strongly UP_i -ideals of fully UP-semigroups, provide the necessary examples and prove its generalizations.

From now on, we shall let A be an f -UP-semigroup $A = (A, \cdot, *, 0)$ unless otherwise specified.

Definition 4.1.1 A subset S of an f -UP-semigroup A is called

- (1) a UP_s -subalgebra of A if S is a UP-subalgebra of $(A, \cdot, 0)$, and S is a subsemigroup of $(A, *)$, and
- (2) a UP_i -subalgebra of A if S is a UP-subalgebra of $(A, \cdot, 0)$, and S is an ideal of $(A, *)$.

We have Theorem 4.1.2, 4.1.13, and 4.1.18 directly from Definition 3.0.24.

Theorem 4.1.2 Every UP_i -subalgebra of A is a UP_s -subalgebra of A .

Example 4.1.3 Let $A = \{0, 1, 2, 3\}$ be a set with two binary operations \cdot and $*$ defined by the following Cayley tables:

\cdot	0	1	2	3	$*$	0	1	2	3
0	0	1	2	3	0	0	0	0	0
1	0	0	2	3	1	0	1	0	0
2	0	1	0	3	2	0	0	2	0
3	0	1	2	0	3	0	3	0	0

Then $A = (A, \cdot, *, 0)$ is an f -UP-semigroup. Let $S = \{0, 1, 2\}$. Then S is a UP_s -subalgebra of A . Since $1 \in S$ and $3 \in A$ but $3 * 1 = 3 \notin S$, we have S is not an ideal of $(A, *)$. Thus S is not a UP_i -subalgebra of A .

Definition 4.1.4 A subset S of an f -UP-semigroup $A = (A, \cdot, *, 0)$ is called

- (1) a *near UP_s -filter* of A if S is a near UP-filter of $(A, \cdot, 0)$, and S is a sub-semigroup of $(A, *)$, and
- (2) a *near UP_i -filter* of A if S is a near UP-filter of $(A, \cdot, 0)$, and S is an ideal of $(A, *)$.

We have Theorem 4.1.5, 4.1.7, 4.1.10, 4.1.12, 4.1.15, 4.1.17, 4.1.20, and 4.1.22 directly from a result quoted in Definition 3.0.23.

Theorem 4.1.5 *Every near UP_s -filter of A is a UP_s -subalgebra of A .*

Example 4.1.6 Let $A = \{0, 1, 2, 3\}$ be a set with two binary operations \cdot and $*$ defined by the following Cayley tables:

\cdot	0	1	2	3	$*$	0	1	2	3
0	0	1	2	3	0	0	0	0	0
1	0	0	1	3	1	0	0	0	0
2	0	0	0	3	2	0	0	0	0
3	0	1	1	0	3	0	0	0	1

Then $A = (A, \cdot, *, 0)$ is an f -UP-semigroup. Let $S = \{0, 2\}$. Then S is a UP_s -subalgebra of A . Since $2 \in S$ but $3 \cdot 2 = 1 \notin S$, we have S is not a near UP-filter of $(A, \cdot, 0)$. Thus S is not a near UP_s -filter of A .

Theorem 4.1.7 *Every near UP_i -filter of A is a UP_i -subalgebra of A .*

In Example 4.1.6, we have S is a UP_i -subalgebra of A . Since S is not a near UP-filter of $(A, \cdot, 0)$, we have S is not a near UP_i -filter of A .

Theorem 4.1.8 *Every near UP_i -filter of A is a near UP_s -filter of A .*

In Example 4.1.3, we have S is a near UP_s -filter of A . Since S is not an ideal of $(A, *)$, we have S is not a near UP_i -filter of A .

Definition 4.1.9 A subset S of an f -UP-semigroup $A = (A, \cdot, *, 0)$ is called

- (1) a UP_s -filter of A if S is a UP-filter of $(A, \cdot, 0)$, and S is a subsemigroup of $(A, *)$, and
- (2) a UP_i -filter of A if S is a UP-filter of $(A, \cdot, 0)$, and S is an ideal of $(A, *)$.

Theorem 4.1.10 *Every UP_s -filter of A is a near UP_s -filter of A .*

Example 4.1.11 Let $A = \{0, 1, 2, 3\}$ be a set with two binary operations \cdot and $*$ defined by the following Cayley tables:

\cdot	0	1	2	3		$*$	0	1	2	3
0	0	1	2	3		0	0	0	0	0
1	0	0	2	3		1	0	0	0	0
2	0	0	0	3		2	0	0	0	0
3	0	0	0	0		3	0	0	0	1

Then $A = (A, \cdot, *, 0)$ is an f -UP-semigroup. Let $S = \{0, 2\}$. Then S is a near UP_s -filter of A . Since $2 \cdot 1 = 0 \in S$ and $2 \in S$ but $1 \notin S$, we have S is not a UP-filter of $(A, \cdot, 0)$. Thus S is not a UP_s -filter of A .

Theorem 4.1.12 *Every UP_i -filter of A is a near UP_i -filter of A .*

In Example 4.1.11, we have S is a near UP_i -filter of A . Since S is not a UP-filter of $(A, \cdot, 0)$, we have S is not a UP_i -filter of A .

Theorem 4.1.13 *Every UP_i -filter of A is a UP_s -filter of A .*

In Example 4.1.3, we have S is a UP_s -filter of A . Since S is not an ideal of $(A, *)$, we have S is not a UP_i -filter of A .

Definition 4.1.14 A subset S of an f -UP-semigroup A is called

- (1) a UP_s -ideal of A if S is a UP-ideal of $(A, \cdot, 0)$, and S is a subsemigroup of $(A, *)$, and
- (2) a UP_i -ideal of A if S is a UP-ideal of $(A, \cdot, 0)$, and S is an ideal of $(A, *)$.

Theorem 4.1.15 Every UP_s -ideal of A is a UP_s -filter of A .

Example 4.1.16 Let $A = \{0, 1, 2, 3\}$ be a set with two binary operations \cdot and $*$ defined by the following Cayley tables:

\cdot	0	1	2	3
0	0	1	2	3
1	0	0	2	2
2	0	1	0	2
3	0	1	0	0

$*$	0	1	2	3
0	0	0	0	0
1	0	0	0	0
2	0	0	0	0
3	0	0	0	0

Then $A = (A, \cdot, *, 0)$ is an f -UP-semigroup. Let $S = \{0, 1\}$. Then S is a UP_s -filter of A . Since $2 \cdot (1 \cdot 3) = 0 \in S$ and $1 \in S$ but $2 \cdot 3 = 2 \notin S$, we have S is not a UP-ideal of $(A, \cdot, 0)$. Thus S is not a UP_s -ideal of A .

Theorem 4.1.17 Every UP_i -ideal of A is a UP_i -filter of A .

In Example 4.1.16, we have S is a UP_i -filter of A . Since S is not a UP-ideal of $(A, \cdot, 0)$, we have S is not a UP_i -ideal of A .

Theorem 4.1.18 Every UP_i -ideal of A is a UP_s -ideal of A .

In Example 4.1.3, we have S is a UP_s -ideal of A . Since S is not an ideal of $(A, *)$, we have S is not a UP_i -ideal of A .

Definition 4.1.19 A subset S of an f -UP-semigroup A is called

- (1) a *strongly UP_s -ideal* of A if S is a strongly UP-ideal of $(A, \cdot, 0)$, and S is a subsemigroup of $(A, *)$, and
- (2) a *strongly UP_i -ideal* of A if S is a strongly UP-ideal of $(A, \cdot, 0)$, and S is an ideal of $(A, *)$.

Theorem 4.1.20 Every strongly UP_s -ideal of A is a UP_s -ideal of A .

Example 4.1.21 Let $A = \{0, 1, 2, 3\}$ be a set with two binary operations \cdot and $*$ defined by the following Cayley tables:

\cdot	0	1	2	3	$*$	0	1	2	3
0	0	1	2	3	0	0	0	0	0
1	0	0	2	3	1	0	0	0	0
2	0	1	0	3	2	0	0	0	1
3	0	1	2	0	3	0	0	1	0

Then $A = (A, \cdot, *, 0)$ is an f -UP-semigroup. Let $S = \{0, 1, 2\}$. Then S is a UP_s -ideal of A . Since $S \neq A$, we have S is not a strongly UP-ideal of $(A, \cdot, 0)$. Thus S is not a strongly UP_s -ideal of A .

Theorem 4.1.22 Every strongly UP_i -ideal of A is a UP_i -ideal of A .

In Example 4.1.21, we have S is a UP_i -ideal of A . Since S is not a strongly UP-ideal of $(A, \cdot, 0)$, we have S is not a strongly UP_i -ideal of A .

Theorem 4.1.23 Strongly UP_s -ideals and strongly UP_i -ideals coincide in A and it is only A .

Proof. It is straightforward by A is the only one strongly UP-ideal of itself. \square

4.2 Fuzzy sets in fully UP-semigroups

In this section, we introduce the notions of fuzzy UP_s -subalgebras, fuzzy UP_i -subalgebras, fuzzy UP_s -filters, fuzzy UP_i -filters, fuzzy UP_s -ideals, fuzzy UP_i -ideals, fuzzy strongly UP_s -ideals, and fuzzy strongly UP_i -ideals of fully UP-semigroups, provide the necessary examples, prove its generalizations and investigate the algebraic properties of fuzzy sets under the operations of intersection and union.

Definition 4.2.1 A fuzzy set F in an f -UP-semigroup A is called

- (1) a *fuzzy UP_s -subalgebra* of A if F is a fuzzy UP-subalgebra of $(A, \cdot, 0)$ and a fuzzy subsemigroup of $(A, *)$, and
- (2) a *fuzzy UP_i -subalgebra* of A if F is a fuzzy UP-subalgebra of $(A, \cdot, 0)$ and a fuzzy ideal of $(A, *)$.

Clearly, a fuzzy UP_i -subalgebra is a fuzzy UP_s -subalgebra.

In Example 4.1.21, we define a membership function f_F as follows:

$$f_F(0) = 1, \quad f_F(1) = 0.4, \quad f_F(2) = 0.5, \quad \text{and} \quad f_F(3) = 0.2.$$

Then F is a fuzzy UP_s -subalgebra of A . Since $f_F(2 * 3) = f_F(1) = 0.4 \not\geq 0.5 = \max\{0.5, 0.2\} = \max\{f_F(2), f_F(3)\}$, we have F is not a fuzzy UP_i -subalgebra of A .

Theorem 4.2.2 *The intersection of any nonempty family of fuzzy UP_s -subalgebras of A is also a fuzzy UP_s -subalgebra of A .*

Proof. Let F_i be a fuzzy UP_s -subalgebra of A for all $i \in I$. Then

$$f_{\bigcap_{i \in I} F_i}(x \cdot y) = \inf\{f_{F_i}(x \cdot y)\}_{i \in I}$$

$$\begin{aligned}
&\geq \inf\{\min\{f_{F_i}(x), f_{F_i}(y)\}\}_{i \in I} \\
&= \min\{\inf\{f_{F_i}(x)\}_{i \in I}, \inf\{f_{F_i}(y)\}_{i \in I}\} \\
&= \min\{f_{\bigcap_{i \in I} F_i}(x), f_{\bigcap_{i \in I} F_i}(y)\} \text{ and} \\
f_{\bigcap_{i \in I} F_i}(x * y) &= \inf\{f_{F_i}(x * y)\}_{i \in I} \\
&\geq \inf\{\min\{f_{F_i}(x), f_{F_i}(y)\}\}_{i \in I} \\
&= \min\{\inf\{f_{F_i}(x)\}_{i \in I}, \inf\{f_{F_i}(y)\}_{i \in I}\} \\
&= \min\{f_{\bigcap_{i \in I} F_i}(x), f_{\bigcap_{i \in I} F_i}(y)\}.
\end{aligned}$$

Hence, $\bigcap_{i \in I} F_i$ is a fuzzy UP_s -subalgebra of A . □

In Example 4.1.21, we define two membership functions f_{F_1} and f_{F_2} as follows:

A	0	1	2	3
f_{F_1}	0.7	0.5	0.7	0.3
f_{F_2}	0.7	0.3	0.2	0.6

Then F_1 and F_2 are fuzzy UP_s -subalgebras of A . Since $f_{F_1 \cup F_2}(3 * 2) = f_{F_1 \cup F_2}(1) = 0.5 \not\geq 0.6 = \min\{0.6, 0.7\} = \min\{f_{F_1 \cup F_2}(3), f_{F_1 \cup F_2}(2)\}$, we have $F_1 \cup F_2$ is not a fuzzy UP_s -subalgebra of A .

Theorem 4.2.3 *A nonempty subset S of A is a UP_s -subalgebra of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy UP_s -subalgebra of A .*

Proof. It is straightforward by Theorem 3.0.31 (1) and Theorem 3.0.29 (1). □

Theorem 4.2.4 *The intersection of any nonempty family of fuzzy UP_i -subalgebras of A is also a fuzzy UP_i -subalgebra of A .*

Proof. Let F_i be a fuzzy UP_i -subalgebra of A for all $i \in I$. Then

$$f_{\bigcap_{i \in I} F_i}(x \cdot y) = \inf\{f_{F_i}(x \cdot y)\}_{i \in I}$$

$$\begin{aligned}
&\geq \inf\{\min\{f_{F_i}(x), f_{F_i}(y)\}\}_{i \in I} \\
&= \min\{\inf\{f_{F_i}(x)\}_{i \in I}, \inf\{f_{F_i}(y)\}_{i \in I}\} \\
&= \min\{f_{\bigcap_{i \in I} F_i}(x), f_{\bigcap_{i \in I} F_i}(y)\} \text{ and} \\
f_{\bigcap_{i \in I} F_i}(x * y) &= \inf\{f_{F_i}(x * y)\}_{i \in I} \\
&\geq \inf\{\max\{f_{F_i}(x), f_{F_i}(y)\}\}_{i \in I} \\
&\geq \max\{\inf\{f_{F_i}(x)\}_{i \in I}, \inf\{f_{F_i}(y)\}_{i \in I}\} \\
&= \max\{f_{\bigcap_{i \in I} F_i}(x), f_{\bigcap_{i \in I} F_i}(y)\}.
\end{aligned}$$

Hence, $\bigcap_{i \in I} F_i$ is a fuzzy UP_i -subalgebra of A . □

In Example 4.1.16, we define two membership functions f_{F_1} and f_{F_2} as follows:

A	0	1	2	3
f_{F_1}	0.9	0.7	0.1	0.1
f_{F_2}	0.8	0.4	0.5	0.6

Then F_1 and F_2 are fuzzy UP_i -subalgebras of A . Since $f_{F_1 \cup F_2}(1 \cdot 3) = f_{F_1 \cup F_2}(2) = 0.5 \not\geq 0.6 = \min\{0.7, 0.6\} = \min\{f_{F_1 \cup F_2}(1), f_{F_1 \cup F_2}(3)\}$, we have $F_1 \cup F_2$ is not a fuzzy UP_i -subalgebra of A .

Theorem 4.2.5 *A nonempty subset S of A is a UP_i -subalgebra of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy UP_i -subalgebra of A .*

Proof. It is straightforward by Theorem 3.0.31 (2) and Theorem 3.0.29 (1). □

Definition 4.2.6 A fuzzy set F in an f -UP-semigroup A is called

- (1) a *fuzzy near UP_s -filter* of A if F is a fuzzy near UP-filter of $(A, \cdot, 0)$ and a fuzzy subsemigroup of $(A, *)$, and
- (2) a *fuzzy near UP_i -filter* of A if F is a fuzzy near UP-filter of $(A, \cdot, 0)$ and a fuzzy ideal of $(A, *)$.

Clearly, a fuzzy near UP_i -filter is a fuzzy near UP_s -filter.

In Example 4.1.21, we define a membership function f_F as follows:

$$f_F(0) = 1, f_F(1) = 0.4, f_F(2) = 0.5, \text{ and } f_F(3) = 0.2.$$

Then F is a fuzzy near UP_s -filter of A . Since $f_F(2 * 3) = f_F(1) = 0.4 \not\geq 0.5 = \max\{0.5, 0.2\} = \max\{f_F(2), f_F(3)\}$, we have F is not a fuzzy near UP_i -filter of A .

Theorem 4.2.7 *The intersection of any nonempty family of fuzzy near UP_s -filters of an f -UP-semigroup $A = (A, \cdot, *, 0)$ is also a fuzzy near UP_s -filter.*

Proof. Let F_i be a fuzzy near UP_s -filter of an f -UP-semigroup $A = (A, \cdot, *, 0)$ for all $i \in I$. Then

$$\begin{aligned} f_{\bigcap_{i \in I} F_i}(0) &= \inf\{f_{F_i}(0)\}_{i \in I} \\ &\geq \inf\{f_{F_i}(x)\}_{i \in I} \\ &= f_{\bigcap_{i \in I} F_i}(x), \\ f_{\bigcap_{i \in I} F_i}(x \cdot y) &= \inf\{f_{F_i}(x \cdot y)\}_{i \in I} \\ &\geq \inf\{f_{F_i}(y)\}_{i \in I} \\ &= f_{\bigcap_{i \in I} F_i}(y), \text{ and} \\ f_{\bigcap_{i \in I} F_i}(x * y) &= \inf\{f_{F_i}(x * y)\}_{i \in I} \\ &\geq \inf\{\min\{f_{F_i}(x), f_{F_i}(y)\}\}_{i \in I} \\ &= \min\{\inf\{f_{F_i}(x)\}_{i \in I}, \inf\{f_{F_i}(y)\}_{i \in I}\} \\ &= \min\{f_{\bigcap_{i \in I} F_i}(x), f_{\bigcap_{i \in I} F_i}(y)\}. \end{aligned}$$

Hence, $\bigcap_{i \in I} F_i$ is a fuzzy near UP_s -filter of A . □

In Example 4.1.21, we define two membership functions f_{F_1} and f_{F_2} as

follows:

A	0	1	2	3
f_{F_1}	1	0.7	1	0.5
f_{F_2}	1	0.5	0.3	0.8

Then F_1 and F_2 are fuzzy near UP_s -filters of A but $F_1 \cup F_2$ is not a fuzzy near UP_s -filter of A . Indeed, $f_{F_1 \cup F_2}(3 * 2) = f_{F_1 \cup F_2}(1) = 0.7 \not\geq 0.8 = \min\{0.8, 1\} = \min\{f_{F_1 \cup F_2}(3), f_{F_1 \cup F_2}(2)\}$.

Theorem 4.2.8 *A nonempty subset S of A is a near UP_s -filter of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy near UP_s -filter of A .*

Proof. It is straightforward by Theorem 3.0.31 (1) and Theorem 3.0.29 (2). \square

Theorem 4.2.9 *The intersection of any nonempty family of fuzzy near UP_i -filters of an f -UP-semigroup $A = (A, \cdot, *, 0)$ is also a fuzzy near UP_i -filter.*

Proof. Let F_i be a fuzzy near UP_i -filter of an f -UP-semigroup $A = (A, \cdot, *, 0)$ for all $i \in I$. Then, by the proof of Theorem 4.2.7, we have $f_{\bigcap_{i \in I} F_i}(0) \geq f_{\bigcap_{i \in I} F_i}(x)$ and $f_{\bigcap_{i \in I} F_i}(x \cdot y) \geq f_{\bigcap_{i \in I} F_i}(y)$. Thus

$$\begin{aligned}
 f_{\bigcap_{i \in I} F_i}(x * y) &= \inf\{f_{F_i}(x * y)\}_{i \in I} \\
 &\geq \inf\{\max\{f_{F_i}(x), f_{F_i}(y)\}\}_{i \in I} \\
 &\geq \max\{\inf\{f_{F_i}(x)\}_{i \in I}, \inf\{f_{F_i}(y)\}_{i \in I}\} \\
 &= \max\{f_{\bigcap_{i \in I} F_i}(x), f_{\bigcap_{i \in I} F_i}(y)\}.
 \end{aligned}$$

Hence, $\bigcap_{i \in I} F_i$ is a fuzzy near UP_i -filter of A . \square

Theorem 4.2.10 *The union of any nonempty family of fuzzy near UP_i -filters of an f -UP-semigroup $A = (A, \cdot, *, 0)$ is also a fuzzy near UP_i -filter.*

Proof. Let F_i be a fuzzy near UP_i -filter of an f -UP-semigroup $A = (A, \cdot, *, 0)$ for

all $i \in I$. Then

$$\begin{aligned}
f_{\bigcup_{i \in I} F_i}(0) &= \sup\{f_{F_i}(0)\}_{i \in I} \\
&\geq \sup\{f_{F_i}(x)\}_{i \in I} \\
&= f_{\bigcup_{i \in I} F_i}(x), \\
f_{\bigcup_{i \in I} F_i}(x \cdot y) &= \sup\{f_{F_i}(x \cdot y)\}_{i \in I} \\
&\geq \sup\{f_{F_i}(y)\}_{i \in I} \\
&= f_{\bigcup_{i \in I} F_i}(y), \text{ and} \\
f_{\bigcup_{i \in I} F_i}(x * y) &= \sup\{f_{F_i}(x * y)\}_{i \in I} \\
&\geq \sup\{\max\{f_{F_i}(x), f_{F_i}(y)\}\}_{i \in I} \\
&= \max\{\sup\{f_{F_i}(x)\}_{i \in I}, \sup\{f_{F_i}(y)\}_{i \in I}\} \\
&= \max\{f_{\bigcup_{i \in I} F_i}(x), f_{\bigcup_{i \in I} F_i}(y)\}.
\end{aligned}$$

Hence, $\bigcup_{i \in I} F_i$ is a fuzzy near UP_i -filter of A . \square

Theorem 4.2.11 *A nonempty subset S of A is a near UP_i -filter of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy near UP_i -filter of A .*

Proof. It is straightforward by Theorem 3.0.31 (2) and Theorem 3.0.29 (2). \square

We have Theorem 4.2.12, 4.2.13, 4.2.20, 4.2.22, 4.2.28, 4.2.29, and 4.2.36 directly from a result quoted in Definition 3.0.26.

Theorem 4.2.12 *Every fuzzy near UP_s -filter of an f - UP -semigroup is a fuzzy UP_s -subalgebra.*

In Example 4.1.6, we define a membership function f_F as follows:

$$f_F(0) = 1, f_F(1) = 0.8, f_F(2) = 0.9, \text{ and } f_F(3) = 0.7.$$

Then F is a fuzzy UP_s -subalgebra of A . Since $f_F(1 \cdot 2) = f_F(1) = 0.8 \not\geq 0.9 = f_F(2)$, we have F is not a fuzzy near UP_s -filter of A .

Theorem 4.2.13 *Every fuzzy near UP_i -filter of an f - UP -semigroup is a fuzzy UP_i -subalgebra.*

In Example 4.1.6, we define a membership function f_F as follows:

$$f_F(0) = 0.8, f_F(1) = 0.4, f_F(2) = 0.8, \text{ and } f_F(3) = 0.3.$$

Then F is a fuzzy UP_i -subalgebra of A . Since $f_F(1 \cdot 2) = f_F(1) = 0.4 \not\geq 0.8 = f_F(2)$, we have F is not a fuzzy near UP_i -filter of A .

Definition 4.2.14 A fuzzy set F in an f - UP -semigroup A is called

- (1) a *fuzzy UP_s -filter* of A if F is a fuzzy UP -filter of $(A, \cdot, 0)$ and a fuzzy subsemigroup of $(A, *)$, and
- (2) a *fuzzy UP_i -filter* of A if F is a fuzzy UP -filter of $(A, \cdot, 0)$ and a fuzzy ideal of $(A, *)$.

Clearly, a fuzzy UP_i -filter is a fuzzy UP_s -filter.

In Example 4.1.21, we define a membership function f_F as follows:

$$f_F(0) = 1, f_F(1) = 0.4, f_F(2) = 0.5, \text{ and } f_F(3) = 0.2.$$

Then F is a fuzzy UP_s -filter of A . Since $f_F(2 * 3) = f_F(1) = 0.4 \not\geq 0.5 = \max\{0.5, 0.2\} = \max\{f_F(2), f_F(3)\}$, we have F is not a fuzzy UP_i -filter of A .

Theorem 4.2.15 *The intersection of any nonempty family of fuzzy UP_s -filters of A is also a fuzzy UP_s -filter of A .*

Proof. Let F_i be a fuzzy UP_s -filter of A for all $i \in I$. Then

$$\begin{aligned}
f_{\bigcap_{i \in I} F_i}(0) &= \inf \{f_{F_i}(0)\}_{i \in I} \\
&\geq \inf \{f_{F_i}(x)\}_{i \in I} \\
&= f_{\bigcap_{i \in I} F_i}(x), \\
f_{\bigcap_{i \in I} F_i}(y) &= \inf \{f_{F_i}(y)\}_{i \in I} \\
&\geq \inf \{\min \{f_{F_i}(x \cdot y), f_{F_i}(x)\}\}_{i \in I} \\
&= \min \{\inf \{f_{F_i}(x \cdot y)\}_{i \in I}, \inf \{f_{F_i}(x)\}_{i \in I}\} \\
&= \min \{f_{\bigcap_{i \in I} F_i}(x \cdot y), f_{\bigcap_{i \in I} F_i}(x)\}, \text{ and} \\
f_{\bigcap_{i \in I} F_i}(x * y) &= \inf \{f_{F_i}(x * y)\}_{i \in I} \\
&\geq \inf \{\min \{f_{F_i}(x), f_{F_i}(y)\}\}_{i \in I} \\
&= \min \{\inf \{f_{F_i}(x)\}_{i \in I}, \inf \{f_{F_i}(y)\}_{i \in I}\} \\
&= \min \{f_{\bigcap_{i \in I} F_i}(x), f_{\bigcap_{i \in I} F_i}(y)\}.
\end{aligned}$$

Hence, $\bigcap_{i \in I} F_i$ is a fuzzy UP_s -filter of A . □

In Example 4.1.21, we define two membership functions f_{F_1} and f_{F_2} as follows:

A	0	1	2	3
f_{F_1}	0.7	0.5	0.7	0.3
f_{F_2}	0.7	0.3	0.2	0.6

Then F_1 and F_2 are fuzzy UP_s -filters of A . Since $f_{F_1 \cup F_2}(2 * 3) = f_{F_1 \cup F_2}(1) = 0.5 \not\geq 0.6 = \min\{0.7, 0.6\} = \min\{f_{F_1 \cup F_2}(2), f_{F_1 \cup F_2}(3)\}$, we have $F_1 \cup F_2$ is not a fuzzy UP_s -filter of A .

Theorem 4.2.16 *A nonempty subset S of A is a UP_s -filter of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy UP_s -filter of A .*

Proof. It is straightforward by Theorem 3.0.31 (1) and Theorem 3.0.29 (3). □

Theorem 4.2.17 *The intersection of any nonempty family of fuzzy UP_i -filters of A is also a fuzzy UP_i -filter of A .*

Proof. Let F_i be a fuzzy UP_i -filter of A for all $i \in I$. Then

$$\begin{aligned}
 f_{\bigcap_{i \in I} F_i}(0) &= \inf \{f_{F_i}(0)\}_{i \in I} \\
 &\geq \inf \{f_{F_i}(x)\}_{i \in I} \\
 &= f_{\bigcap_{i \in I} F_i}(x), \\
 f_{\bigcap_{i \in I} F_i}(y) &= \inf \{f_{F_i}(y)\}_{i \in I} \\
 &\geq \inf \{\min \{f_{F_i}(x \cdot y), f_{F_i}(x)\}\}_{i \in I} \\
 &= \min \{\inf \{f_{F_i}(x \cdot y)\}_{i \in I}, \inf \{f_{F_i}(x)\}_{i \in I}\} \\
 &= \min \{f_{\bigcap_{i \in I} F_i}(x \cdot y), f_{\bigcap_{i \in I} F_i}(x)\}, \text{ and} \\
 f_{\bigcap_{i \in I} F_i}(x * y) &= \inf \{f_{F_i}(x * y)\}_{i \in I} \\
 &\geq \inf \{\max \{f_{F_i}(x), f_{F_i}(y)\}\}_{i \in I} \\
 &\geq \max \{\inf \{f_{F_i}(x)\}_{i \in I}, \inf \{f_{F_i}(y)\}_{i \in I}\} \\
 &= \max \{f_{\bigcap_{i \in I} F_i}(x), f_{\bigcap_{i \in I} F_i}(y)\}.
 \end{aligned}$$

Hence, $\bigcap_{i \in I} F_i$ is a fuzzy UP_i -filter of A . □

Example 4.2.18 Let $A = \{0, 1, 2, 3\}$ be a set with two binary operations \cdot and $*$ defined by the following Cayley tables:

\cdot	0	1	2	3	$*$	0	1	2	3
0	0	1	2	3	0	0	0	0	0
1	0	0	2	2	1	0	0	0	0
2	0	1	0	1	2	0	0	0	0
3	0	0	0	0	3	0	0	0	0

Then $A = (A, \cdot, *, 0)$ is an f -UP-semigroup. We define two membership functions

f_{F_1} and f_{F_2} as follows:

A	0	1	2	3
f_{F_1}	0.9	0.9	0.5	0.5
f_{F_2}	1	0.5	0.6	0.5

Then F_1 and F_2 are fuzzy UP_i -filters of A . Since $f_{F_1 \cup F_2}(3) = 0.5 \not\geq 0.6 = \min\{0.9, 0.6\} = \min\{f_{F_1 \cup F_2}(1), f_{F_1 \cup F_2}(2)\} = \min\{f_{F_1 \cup F_2}(2 \cdot 3), f_{F_1 \cup F_2}(2)\}$, we have $F_1 \cup F_2$ is not a fuzzy UP_i -filter of A .

Theorem 4.2.19 *A nonempty subset S of A is a UP_i -filter of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy UP_i -filter of A .*

Proof. It is straightforward by Theorem 3.0.31 (2) and Theorem 3.0.29 (3). \square

Theorem 4.2.20 *Every fuzzy UP_s -filter of an f -UP-semigroup is a fuzzy near UP_s -filter.*

The following example shows that the converse of Theorem 4.2.20 is not true.

Example 4.2.21 Let $A = \{0, 1, 2, 3\}$ be a set with two binary operations \cdot and $*$ defined by the following Cayley tables:

\cdot	0	1	2	3	$*$	0	1	2	3
0	0	1	2	3	0	0	0	0	0
1	0	0	2	3	1	0	0	0	0
2	0	0	0	3	2	0	0	0	0
3	0	0	0	0	3	0	0	0	2

Then $A = (A, \cdot, *, 0)$ is an f -UP-semigroup. We define a membership function f_F as follows:

$$f_F(0) = 1, f_F(1) = 0.7, f_F(2) = 0.9, \text{ and } f_F(3) = 0.8.$$

Then F is a fuzzy near UP_s -filter of A . Since $f_F(1) = 0.7 \not\geq 0.8 = \min\{1, 0.8\} = \min\{f_F(0), f_F(3)\} = \min\{f_F(3 \cdot 1), f_F(3)\}$, we have F is not a fuzzy UP_s -filter of A .

Theorem 4.2.22 *Every fuzzy UP_i -filter of an f -UP-semigroup is a fuzzy near UP_i -filter.*

In Example 4.2.21, we have F is a fuzzy near UP_i -filter of A but it is not a fuzzy UP_i -filter of A .

Definition 4.2.23 A fuzzy set F in an f -UP-semigroup A is called

- (1) a *fuzzy UP_s -ideal* of A if F is a fuzzy UP-ideal of $(A, \cdot, 0)$ and a fuzzy subsemigroup of $(A, *)$, and
- (2) a *fuzzy UP_i -ideal* of A if F is a fuzzy UP-ideal of $(A, \cdot, 0)$ and a fuzzy ideal of $(A, *)$.

Clearly, a fuzzy UP_i -ideal is a fuzzy UP_s -ideal.

In Example 4.1.21, we define a membership function f_F as follows:

$$f_F(0) = 1, f_F(1) = 0.4, f_F(2) = 0.5, \text{ and } f_F(3) = 0.2.$$

Then F is a fuzzy UP_s -ideal of A . Since $f_F(3 * 2) = f_F(1) = 0.4 \not\geq 0.5 = \max\{0.2, 0.5\} = \max\{f_F(3), f_F(2)\}$, we have F is not a fuzzy UP_i -ideal of A .

Theorem 4.2.24 *The intersection of any nonempty family of fuzzy UP_s -ideals of A is also a fuzzy UP_s -ideal of A .*

Proof. Let F_i be a fuzzy UP_s -ideal of A for all $i \in I$. Then

$$f_{\bigcap_{i \in I} F_i}(0) = \inf\{f_{F_i}(0)\}_{i \in I}$$

$$\begin{aligned}
&\geq \inf\{f_{F_i}(x)\}_{i \in I} \\
&= f_{\bigcap_{i \in I} F_i}(x), \\
f_{\bigcap_{i \in I} F_i}(x \cdot z) &= \inf\{f_{F_i}(x \cdot z)\}_{i \in I} \\
&\geq \inf\{\min\{f_{F_i}(x \cdot (y \cdot z)), f_{F_i}(y)\}\}_{i \in I} \\
&= \min\{\inf\{f_{F_i}(x \cdot (y \cdot z))\}_{i \in I}, \inf\{f_{F_i}(y)\}_{i \in I}\} \\
&= \min\{f_{\bigcap_{i \in I} F_i}(x \cdot (y \cdot z)), f_{\bigcap_{i \in I} F_i}(y)\}, \text{ and} \\
f_{\bigcap_{i \in I} F_i}(x * y) &= \inf\{f_{F_i}(x * y)\}_{i \in I} \\
&\geq \inf\{\min\{f_{F_i}(x), f_{F_i}(y)\}\}_{i \in I} \\
&= \min\{\inf\{f_{F_i}(x)\}_{i \in I}, \inf\{f_{F_i}(y)\}_{i \in I}\} \\
&= \min\{f_{\bigcap_{i \in I} F_i}(x), f_{\bigcap_{i \in I} F_i}(y)\}.
\end{aligned}$$

Hence, $\bigcap_{i \in I} F_i$ is a fuzzy UP_s -ideal of A . □

In Example 4.1.21, we define two membership functions f_{F_1} and f_{F_2} as follows:

A	0	1	2	3
f_{F_1}	0.7	0.5	0.7	0.3
f_{F_2}	0.7	0.3	0.2	0.6

Then F_1 and F_2 are fuzzy UP_s -ideals of A . Since $f_{F_1 \cup F_2}(3 * 2) = f_{F_1 \cup F_2}(1) = 0.5 \not\geq 0.6 = \min\{0.6, 0.7\} = \min\{f_{F_1 \cup F_2}(3), f_{F_1 \cup F_2}(2)\}$, we have $F_1 \cup F_2$ is not a fuzzy UP_s -ideal of A .

Theorem 4.2.25 *A nonempty subset S of A is a UP_s -ideal of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy UP_s -ideal of A .*

Proof. It is straightforward by Theorem 3.0.31 (1) and Theorem 3.0.29 (4). □

Theorem 4.2.26 *The intersection of any nonempty family of fuzzy UP_1 -ideals of A is also a fuzzy UP_1 -ideal of A .*

Proof. Let F_i be a fuzzy UP_i -ideal of A for all $i \in I$. Then

$$\begin{aligned}
f_{\bigcap_{i \in I} F_i}(0) &= \inf \{f_{F_i}(0)\}_{i \in I} \\
&\geq \inf \{f_{F_i}(x)\}_{i \in I} \\
&= f_{\bigcap_{i \in I} F_i}(x), \\
f_{\bigcap_{i \in I} F_i}(x \cdot z) &= \inf \{f_{F_i}(x \cdot z)\}_{i \in I} \\
&\geq \inf \{\min \{f_{F_i}(x \cdot (y \cdot z)), f_{F_i}(y)\}\}_{i \in I} \\
&= \min \{\inf \{f_{F_i}(x \cdot (y \cdot z))\}_{i \in I}, \inf \{f_{F_i}(y)\}_{i \in I}\} \\
&= \min \{f_{\bigcap_{i \in I} F_i}(x \cdot (y \cdot z)), f_{\bigcap_{i \in I} F_i}(y)\}, \text{ and} \\
f_{\bigcap_{i \in I} F_i}(x * y) &= \inf \{f_{F_i}(x * y)\}_{i \in I} \\
&\geq \inf \{\max \{f_{F_i}(x), f_{F_i}(y)\}\}_{i \in I} \\
&\geq \max \{\inf \{f_{F_i}(x)\}_{i \in I}, \inf \{f_{F_i}(y)\}_{i \in I}\} \\
&= \max \{f_{\bigcap_{i \in I} F_i}(x), f_{\bigcap_{i \in I} F_i}(y)\}.
\end{aligned}$$

Hence, $\bigcap_{i \in I} F_i$ is a fuzzy UP_i -ideal of A . □

In Example 4.2.18, we define two membership functions f_{F_1} and f_{F_2} as follows:

A	0	1	2	3
f_{F_1}	0.7	0.3	0.4	0.3
f_{F_2}	0.8	0.5	0.2	0.2

Then F_1 and F_2 are fuzzy UP_i -ideals of A . Since $f_{F_1 \cup F_2}(0 \cdot 3) = f_{F_1 \cup F_2}(3) = 0.3 \neq 0.4 = \min \{0.4, 0.5\} = \min \{f_{F_1 \cup F_2}(2), f_{F_1 \cup F_2}(1)\} = \min \{f_{F_1 \cup F_2}(0 \cdot (1 \cdot 3)), f_{F_1 \cup F_2}(1)\}$, we have $F_1 \cup F_2$ is not a fuzzy UP_i -ideal of A .

Theorem 4.2.27 *A nonempty subset S of A is a UP_i -ideal of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy UP_i -ideal of A .*

Proof. It is straightforward by Theorem 3.0.31 (2) and Theorem 3.0.29 (4). □

Theorem 4.2.28 *Every fuzzy UP_s -ideal of A is a fuzzy UP_s -filter of A .*

In Example 4.1.16, we define a membership function f_F as follows:

$$f_F(0) = 0.8, f_F(1) = 0.6, f_F(2) = 0.3, \text{ and } f_F(3) = 0.3.$$

Then F is a fuzzy UP_s -filter of A . Since $f_F(2 \cdot 3) = f_F(2) = 0.3 \not\geq 0.6 = \min\{0.8, 0.6\} = \min\{f_F(0), f_F(1)\} = \min\{f_F(2 \cdot (1 \cdot 3)), f_F(1)\}$, we have F is not a fuzzy UP_s -ideal of A .

Theorem 4.2.29 *Every fuzzy UP_i -ideal of A is a fuzzy UP_i -filter of A .*

In Example 4.1.16, we define a membership function f_F as follows:

$$f_F(0) = 0.8, f_F(1) = 0.6, f_F(2) = 0.3, \text{ and } f_F(3) = 0.3.$$

Then F is a fuzzy UP_i -filter of A . Since $f_F(2 \cdot 3) = f_F(2) = 0.3 \not\geq 0.6 = \max\{0.8, 0.6\} = \max\{f_F(0), f_F(1)\} = \max\{f_F(2 \cdot (1 \cdot 3)), f_F(1)\}$, we have F is not a fuzzy UP_i -ideal of A .

Definition 4.2.30 A fuzzy set F in an f -UP-semigroup A is called

- (1) a *fuzzy strongly UP_s -ideal* of A if F is a fuzzy strongly UP -ideal of $(A, \cdot, 0)$ and a fuzzy subsemigroup of $(A, *)$, and
- (2) a *fuzzy strongly UP_i -ideal* of A if F is a fuzzy strongly UP -ideal of $(A, \cdot, 0)$ and a fuzzy ideal of $(A, *)$.

Theorem 4.2.31 *Fuzzy strongly UP_s -ideals, fuzzy strongly UP_i -ideals, and constant fuzzy sets coincide in A .*

Proof. It is straightforward by Theorem 3.0.28. □

If a fuzzy set F_i is constant for all $i \in I$, then we see that the fuzzy sets $\bigcap_{i \in I} F_i$ and $\bigvee_{i \in I} F_i$ are constant. From this, we have Theorem 4.2.32 and 4.2.33.

Theorem 4.2.32 *The intersection and union of any nonempty family of fuzzy strongly UP_s -ideals of A are also a fuzzy strongly UP_s -ideal of A .*

Theorem 4.2.33 *The intersection and union of any nonempty family of fuzzy strongly UP_i -ideals of A are also a fuzzy strongly UP_i -ideal of A .*

Theorem 4.2.34 *A nonempty subset S of A is a strongly UP_s -ideal of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy strongly UP_s -ideal of A .*

Proof. It is straightforward by Theorem 3.0.31 (1) and Theorem 3.0.29 (5). \square

Theorem 4.2.35 *A nonempty subset S of A is a strongly UP_i -ideal of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy strongly UP_i -ideal of A .*

Proof. It is straightforward by Theorem 3.0.31 (2) and Theorem 3.0.29 (5). \square

Theorem 4.2.36 *Every fuzzy strongly UP_s -ideal (fuzzy strongly UP_i -ideal) of A is a fuzzy UP_s -ideal and a fuzzy UP_i -ideal of A .*

In Example 4.1.3, we define a membership function f_F as follows:

$$f_F(0) = 0.7, f_F(1) = 0.5, f_F(2) = 0.3, \text{ and } f_F(3) = 0.6.$$

Then F is a fuzzy UP_i -ideal of A . Since F is not constant, we have F is not a fuzzy strongly UP_s -ideal and a fuzzy strongly UP_i -ideal of A .

Then we get the diagram of generalization of fuzzy sets in fully UP -semigroups as shown in Figure 4.2 below.

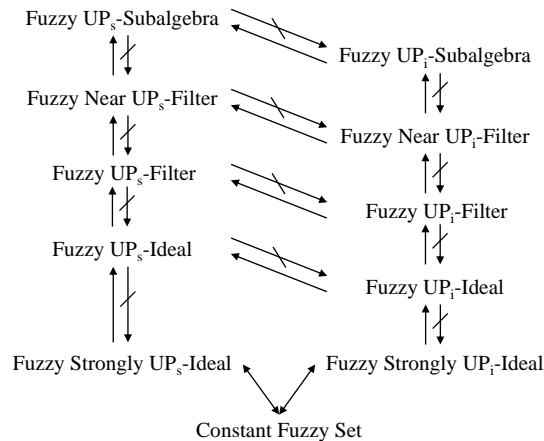


Figure 1: Fuzzy sets in fully UP-semigroups

4.3 Properties of fuzzy sets in UP-algebras

In this section, we shall let A be a UP-algebra $A = (A, \cdot, 0)$ and find some properties of fuzzy sets in UP-algebras.

Proposition 4.3.1 [34] *If F is a fuzzy UP-subalgebra of A , then*

$$(\forall x \in A)(f_F(0) \geq f_F(x)). \quad (4.3.1)$$

Proposition 4.3.2 *If F is a fuzzy UP-filter of A , then*

$$(\forall x, y \in A)(x \leq y \Rightarrow f_F(x) \leq f_F(y)). \quad (4.3.2)$$

Proposition 4.3.3 *If F is a fuzzy set in A satisfying the condition*

$$(\forall x, y, z \in A)(z \leq x \Rightarrow f_F(x \cdot y) \geq \min\{f_F(z), f_F(y)\}), \quad (4.3.3)$$

then F is a fuzzy UP-subalgebra of A .

Proof. Let $x, y \in A$. By (3.0.1), we have $x \leq x$. It follows from (4.3.3) that $f_F(x \cdot y) \geq \min\{f_F(x), f_F(y)\}$. Hence, F is a fuzzy UP-subalgebra of A . \square

Theorem 4.3.4 *If F is a fuzzy set in A satisfying the condition (4.3.3), then F satisfies the condition (4.3.1).*

Proof. It is straightforward by Proposition 4.3.3. \square

The following example shows that the converse of Theorem 4.3.4 is not true.

Example 4.3.5 Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation \cdot defined by the following Cayley table:

\cdot	0	1	2	3
0	0	1	2	3
1	0	0	2	2
2	0	1	0	2
3	0	1	0	0

Then $A = (A, \cdot, 0)$ is a UP-algebra. We define a membership function f_F as follows:

$$f_F(0) = 1, f_F(1) = 0.6, f_F(2) = 0.2, \text{ and } f_F(3) = 0.9.$$

Then F satisfies the condition (4.3.1) but it does not satisfy the condition (4.3.3). Indeed, $1 \leq 1$ but $f_F(1 \cdot 3) = f_F(2) = 0.2 \not\geq 0.6 = \min\{0.6, 0.9\} = \min\{f_F(1), f_F(3)\}$.

It is clear that we have the following proposition.

Proposition 4.3.6 *If F is a fuzzy set in A satisfying the condition*

$$(\forall x, y, z \in A)(f_F(x \cdot y) \geq \min\{f_F(z), f_F(y)\}), \quad (4.3.4)$$

then F satisfies the condition (4.3.3).

The following example shows that the converse of Proposition 4.3.6 is not true.

Example 4.3.7 Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation \cdot defined by the following Cayley table:

\cdot	0	1	2	3
0	0	1	2	3
1	0	0	3	3
2	0	1	0	0
3	0	1	2	0

Then $A = (A, \cdot, 0)$ is a UP-algebra. We define a membership function f_F as follows:

$$f_F(0) = 1, f_F(1) = 0.1, f_F(2) = 0.8, \text{ and } f_F(3) = 0.2.$$

Then F satisfies the condition (4.3.3) but it does not satisfy the condition (4.3.4). Indeed, $f_F(1 \cdot 2) = f_F(3) = 0.2 \not\geq 0.8 = \min\{1, 0.8\} = \min\{f_F(0), f_F(2)\}$.

Proposition 4.3.8 *If F is a fuzzy set in A satisfying the condition (4.3.2), then F is a fuzzy near UP-filter of A .*

Proof. Let $x, y \in A$. By (UP-3), we have $x \leq 0$. It follows from (4.3.2) that $f_F(0) \geq f_F(x)$. By (3.0.5), we have $y \leq x \cdot y$. It follows from (4.3.2) that $f_F(x \cdot y) \geq f_F(y)$. Hence, F is a fuzzy near UP-filter of A . \square

Theorem 4.3.9 *If F is a fuzzy set in A satisfying the condition (4.3.2), then F satisfies the condition (4.3.4).*

Proof. Let $x, y, z \in A$. By (3.0.5), we have $y \leq x \cdot y$. It follows from (4.3.2) that $f_F(x \cdot y) \geq f_F(y) \geq \min\{f_F(z), f_F(y)\}$. Hence, F satisfies (4.3.4). \square

The following example shows that the converse of Theorem 4.3.9 is not true.

Example 4.3.10 Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation \cdot defined by the following Cayley table:

\cdot	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	0	0	3
3	0	0	0	0

Then $A = (A, \cdot, 0)$ is a UP-algebra. We define a membership function f_F as follows:

$$f_F(0) = 1, f_F(1) = 0.1, f_F(2) = 0.7, \text{ and } f_F(3) = 0.8.$$

Then F satisfies the condition (4.3.4) but it does not satisfy the condition (4.3.2). Indeed, $3 \leq 2$ but $f_F(2) = f_F(1) = 0.7 \not\geq 0.8 = f_F(3)$.

Theorem 4.3.11 *If F is a fuzzy UP-subalgebra of A satisfying the condition*

$$(\forall x, y \in A)(x \cdot y \neq 0 \Rightarrow f_F(x) \geq f_F(y)), \quad (4.3.5)$$

then F is a fuzzy near UP-filter of A .

Proof. Let $x, y \in A$. If $x \cdot y = 0$, then by (4.3.1), we have $f_F(x \cdot y) = f_F(0) \geq f_F(y)$. If $x \cdot y \neq 0$, then by (4.3.5), we have $f_F(x \cdot y) \geq \min\{f_F(x), f_F(y)\} = f_F(y)$. Hence, F is a fuzzy near UP-filter of A . \square

Proposition 4.3.12 *A fuzzy set F in A satisfies the condition*

$$(\forall x, y, z \in A)(z \leq x \cdot y \Rightarrow f_F(y) \geq \min\{f_F(z), f_F(x)\}) \quad (4.3.6)$$

if and only if F is a fuzzy UP-filter of A .

Proof. Let $x \in A$. By (UP-3), we have $x \leq x \cdot 0$. It follows from (4.3.6) that $f_F(0) \geq \min\{f_F(x), f_F(x)\} = f_F(x)$. Let $x, y \in A$. By (3.0.1), we have $x \cdot y \leq x \cdot y$. It follows from (4.3.6) that $f_F(y) \geq \min\{f_F(x \cdot y), f_F(x)\}$. Hence, F is a fuzzy UP-filter of A .

Conversely, let $x, y, z \in A$ be such that $z \leq x \cdot y$. Then $z \cdot (x \cdot y) = 0$, so

$$f_F(x \cdot y) \geq \min\{f_F(z \cdot (x \cdot y)), f_F(z)\} = \min\{f_F(0), f_F(z)\} = f_F(z).$$

Thus $f_F(y) \geq \min\{f_F(x \cdot y), f_F(x)\} \geq \min\{f_F(z), f_F(x)\}$. Hence, F satisfies (4.3.6). \square

Theorem 4.3.13 *If F is a fuzzy set in A satisfying the condition (4.3.6), then F satisfies the condition (4.3.2).*

Proof. Let $x, y \in A$ such that $x \leq y$. By (3.0.11), we have $x \leq x \cdot y$. It follows from (4.3.6) that $f_F(y) \geq \min\{f_F(x), f_F(x)\} = f_F(x)$. Hence, F satisfies (4.3.2). \square

The following example shows that the converse of Theorem 4.3.13 is not true.

Example 4.3.14 Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation \cdot defined by the following Cayley table:

\cdot	0	1	2	3
0	0	1	2	3
1	0	0	2	2
2	0	1	0	1
3	0	0	0	0

Then $A = (A, \cdot, 0)$ is a UP-algebra. We define a membership function f_F as

follows:

$$f_F(0) = 0.9, f_F(1) = 0.3, f_F(2) = 0.6, \text{ and } f_F(3) = 0.2.$$

Then F satisfies the condition (4.3.2) but it does not satisfy the condition (4.3.6).

Indeed, $1 \leq 2 \cdot 3$ but $f_F(3) = 0.2 \not\geq 0.3 = \min\{0.3, 0.6\} = \min\{f_F(1), f_F(2)\}$.

Theorem 4.3.15 *If F is a fuzzy near UP-filter of A satisfying the condition*

$$(\forall x, y \in A)(f_F(x \cdot y) = f_F(y)), \quad (4.3.7)$$

then F is a fuzzy UP-filter of A .

Proof. Let $x, y \in A$. By (4.3.7), we have $f_F(y) \geq \min\{f_F(y), f_F(x)\} = \min\{f_F(x \cdot y), f_F(x)\}$. Hence, F is a fuzzy UP-filter of A . \square

Proposition 4.3.16 *A fuzzy set F in A satisfies the condition*

$$(\forall a, x, y, z \in A)(a \leq x \cdot (y \cdot z) \Rightarrow f_F(x \cdot z) \geq \min\{f_F(a), f_F(y)\}) \quad (4.3.8)$$

if and only if F is a fuzzy UP-ideal of A .

Proof. Let $x \in A$. By (UP-3), we have $x \leq x \cdot (x \cdot 0)$. By (UP-3) and (4.3.8), we have

$$f_F(0) = f_F(x \cdot 0) \geq \min\{f_F(x), f_F(x)\} = f_F(x).$$

Let $x, y, z \in A$. By (3.0.1), we have $x \cdot (y \cdot z) \leq x \cdot (y \cdot z)$. It follows from (4.3.8) that

$$f_F(x \cdot z) \geq \min\{f_F(x \cdot (y \cdot z)), f_F(y)\}.$$

Hence, F is a fuzzy UP-ideal of A .

Conversely, let $a, x, y, z \in A$ be such that $a \leq x \cdot (y \cdot z)$. By Proposition 4.3.2, we have $f_F(a) \leq f_F(x \cdot (y \cdot z))$. Thus

$$f_F(x \cdot z) \geq \min\{f_F(x \cdot (y \cdot z)), f_F(y)\} \geq \min\{f_F(a), f_F(y)\}.$$

Hence, F satisfies (4.3.8). □

Proposition 4.3.17 *If F is a fuzzy UP-ideal of A , then*

$$(\forall a, x, y, z \in A)(a \leq x \cdot (y \cdot z) \Rightarrow f_F(a \cdot z) \geq \min\{f_F(x), f_F(y)\}). \quad (4.3.9)$$

Proof. Let $a, x, y, z \in A$ be such that $a \leq x \cdot (y \cdot z)$. Then $a \cdot (x \cdot (y \cdot z)) = 0$, so

$$f_F(a \cdot (y \cdot z)) \geq \min\{f_F(a \cdot (x \cdot (y \cdot z))), f_F(x)\} = \min\{f_F(0), f_F(x)\} = f_F(x).$$

Thus

$$f_F(a \cdot z) \geq \min\{f_F(a \cdot (y \cdot z)), f_F(y)\} \geq \min\{f_F(x), f_F(y)\}.$$

□

Corollary 4.3.18 *If F is a fuzzy set in A satisfying the condition (4.3.8), then F satisfies the condition (4.3.9).*

Proof. It is straightforward by Propositions 4.3.16 and 4.3.17. □

Theorem 4.3.19 *Let A be a UP-algebra satisfying the condition*

$$(\forall x, y, z \in A)(z \cdot (y \cdot x) = y \cdot (z \cdot x)). \quad (4.3.10)$$

If F is a fuzzy set in A satisfying the condition (4.3.9), then F satisfies the condition (4.3.8).

Proof. Let $a, x, y, z \in A$ such that $a \leq x \cdot (y \cdot z)$. By (4.3.10), we have $0 =$

$a \cdot (x \cdot (y \cdot z)) = x \cdot (a \cdot (y \cdot z))$, that is, $x \leq a \cdot (y \cdot z)$. It follows from (4.3.9) that $f_F(x \cdot z) \geq \min\{f_F(a), f_F(y)\}$. Hence, F satisfies (4.3.8). \square

Theorem 4.3.20 *If F is a fuzzy set in A satisfying the condition (4.3.9), then F satisfies the condition (4.3.6).*

Proof. Let $x, y, z \in A$ be such that $z \leq x \cdot y$. By (3.0.1) and (3.0.3), we have $0 = z \cdot z \leq z \cdot (x \cdot y)$. By (UP-2) and (4.3.9), we have $f_F(y) = f_F(0 \cdot y) \geq \min\{f_F(z), f_F(x)\}$. Hence, F satisfies (4.3.6). \square

Corollary 4.3.21 *If F is a fuzzy set in A satisfying the condition (4.3.8), then F satisfies the condition (4.3.6).*

Proof. It is straightforward by Corollary 4.3.18 and Theorem 4.3.20. \square

The following example shows that the converse of Theorem 4.3.20 is not true.

Example 4.3.22 Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation \cdot defined by the following Cayley table:

\cdot	0	1	2	3
0	0	1	2	3
1	0	0	3	3
2	0	1	0	0
3	0	1	2	0

Then $A = (A, \cdot, 0)$ is a UP-algebra. We define a membership function f_F as follows:

$$f_F(0) = 1, f_F(1) = 0.9, f_F(2) = 0.1, \text{ and } f_F(3) = 0.1.$$

Then F satisfies the condition (4.3.6) but it does not satisfy the condition (4.3.9). Indeed, $3 \leq 1 \cdot (1 \cdot 2)$ but $f_F(3 \cdot 2) = f_F(2) = 0.1 \not\geq 0.9 = f_F(1) = \min\{f_F(1), f_F(1)\}$.

The following example shows that fuzzy set in a UP-algebra which satisfies the condition (4.3.8) is not constant.

Example 4.3.23 Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation \cdot defined by the following Cayley table:

\cdot	0	1	2	3
0	0	1	2	3
1	0	0	2	3
2	0	1	0	3
3	0	1	2	0

Then $A = (A, \cdot, 0)$ is a UP-algebra. We define a membership function f_F as follows:

$$f_F(0) = 0.7, f_F(1) = 0.5, f_F(2) = 0.4, \text{ and } f_F(3) = 0.4.$$

Then F satisfies the condition (4.3.8) but it is not constant.

Theorem 4.3.24 *If F is a fuzzy UP-filter of A satisfying the condition*

$$(\forall x, y, z \in A)(f_F(y \cdot (x \cdot z)) = f_F(x \cdot (y \cdot z))), \quad (4.3.11)$$

then F is a fuzzy UP-ideal of A .

Proof. Let $x, y, z \in A$. By (4.3.11), we have

$$f_F(x \cdot z) \geq \min\{f_F(y \cdot (x \cdot z)), f_F(y)\} = \min\{f_F(x \cdot (y \cdot z)), f_F(y)\}.$$

Hence, F is a fuzzy UP-ideal of A . □

Proposition 4.3.25 *A fuzzy set F in A satisfies the condition*

$$(\forall a, x, y, z \in A)(a \leq (z \cdot y) \cdot (z \cdot x) \Rightarrow f_F(x) \geq \min\{f_F(a), f_F(y)\}) \quad (4.3.12)$$

if and only if F is a fuzzy strongly UP-ideal of A .

Proof. Let $x \in A$. By (UP-3), we have $x \leq 0 = x \cdot 0 = (0 \cdot x) \cdot (0 \cdot 0)$. By (4.3.12), we have $f_F(0) \geq \min\{f_F(x), f_F(x)\} = f_F(x)$. Let $x, y, z \in A$. By (3.0.1), we have $(z \cdot y) \cdot (z \cdot x) \leq (z \cdot y) \cdot (z \cdot x)$. By (4.3.12), we have $f_F(x) \geq \min\{f_F((z \cdot y) \cdot (z \cdot x)), f_F(y)\}$. Hence, F is a fuzzy strongly UP-ideal of A .

The converse is obvious because F is constant. \square

Theorem 4.3.26 *If F is a fuzzy set in A satisfying the condition*

$$(\forall x, y, z \in A)(z \leq x \cdot y \Rightarrow f_F(z) \geq \min\{f_F(x), f_F(y)\}), \quad (4.3.13)$$

then F satisfies the condition (4.3.3).

Proof. Let $x, y, z \in A$ be such that $z \leq x$. By (3.0.4), we have $x \cdot y \leq z \cdot y$. By (4.3.13), we have $f_F(x \cdot y) \geq \min\{f_F(z), f_F(y)\}$. Hence, F satisfies (4.3.3). \square

Proposition 4.3.27 *A fuzzy set F in A satisfies the condition (4.3.13) if and only if F is a fuzzy strongly UP-ideal of A .*

Proof. Let $x \in A$. By (UP-3), we have $x \leq 0 = 0 \cdot 0$. By (4.3.13), we have $f_F(x) \geq \min\{f_F(0), f_F(0)\} = f_F(0)$. By Theorem 4.3.26 and Proposition 4.3.3, we have $f_F(0) \geq f_F(x)$. Thus $f_F(x) = f_F(0)$ for all $x \in A$, so F is constant. Hence, F is a fuzzy strongly UP-ideal of A .

The converse is obvious because F is constant. \square

Theorem 4.3.28 *If F is a fuzzy set in A satisfying the condition*

$$(\forall x, y, z \in A)(z \leq x \cdot y \Rightarrow f_F(z) \geq f_F(y)), \quad (4.3.14)$$

then F satisfies the condition (4.3.3).

Proof. Let $x, y, z \in A$ be such that $z \leq x$. By (3.0.4), we have $x \cdot y \leq z \cdot y$. It follows from (4.3.14) that $f_F(x \cdot y) \geq f_F(y) \geq \min\{f_F(z), f_F(y)\}$. Hence, F satisfies (4.3.3). \square

Proposition 4.3.29 *A fuzzy set F in A satisfies the condition (4.3.14) if and only if F is a fuzzy strongly UP-ideal of A .*

Proof. Let $x \in A$. By (UP-3), we have $x \leq 0 = 0 \cdot 0$. By (4.3.14), we have $f_F(x) \geq f_F(0)$. By Theorem 4.3.28 and Proposition 4.3.3, we have $f_F(0) \geq f_F(x)$. Thus $f_F(x) = f_F(0)$ for all $x \in A$, so F is constant. Hence, F is a fuzzy strongly UP-ideal of A .

The converse is obvious because F is constant. \square

We have provided various important properties of fuzzy sets in various types in UP-algebras which will be used in the next section. We get the diagram of the properties of fuzzy sets in UP-algebras as shown in Figure 4.3 below.

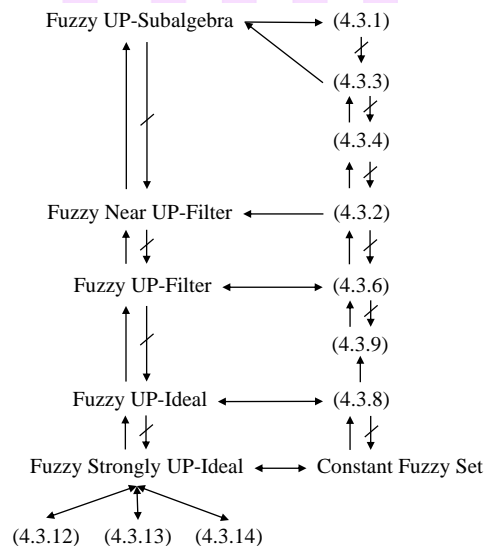


Figure 2: Properties of fuzzy sets in UP-algebras

4.4 Fuzzy soft sets over fully UP-semigroups

From now on, we shall let A be an f -UP-semigroup $A = (A, \cdot, *, 0)$ and P be a set of parameters. Let $\mathcal{F}(A)$ denotes the set of all fuzzy sets in A . A subset E of P is called a *set of statistics*.

Definition 4.4.1 Let $E \subseteq P$. A pair (\tilde{F}, E) is called a *fuzzy soft set* over A if \tilde{F} is a mapping given by $\tilde{F}: E \rightarrow \mathcal{F}(A)$, that is, a fuzzy soft set is a statistic family of fuzzy sets in A . In general, for every $e \in E$, $\tilde{F}[e] := \{(x, f_{\tilde{F}[e]}(x)) \mid x \in A\}$ is a fuzzy set in A and it is called a *fuzzy value set* of statistic e .

Definition 4.4.2 Let (\tilde{F}, E_1) and (\tilde{G}, E_2) be two fuzzy soft sets over a common universe U . The *union* [24] of (\tilde{F}, E_1) and (\tilde{G}, E_2) is defined to be the fuzzy soft set $(\tilde{F}, E_1) \cup (\tilde{G}, E_2) = (\tilde{H}, E)$ satisfying the following conditions:

- (i) $E = E_1 \cup E_2$ and
 - (ii) for all $e \in E$,
- $$\tilde{H}[e] = \begin{cases} \tilde{F}[e] & \text{if } e \in E_1 \setminus E_2 \\ \tilde{G}[e] & \text{if } e \in E_2 \setminus E_1 \\ \tilde{F}[e] \cup \tilde{G}[e] & \text{if } e \in E_1 \cap E_2. \end{cases}$$

The *restricted union* [28] of (\tilde{F}, E_1) and (\tilde{G}, E_2) is defined to be the fuzzy soft set $(\tilde{F}, E_1) \uplus (\tilde{G}, E_2) = (\tilde{H}, E)$ satisfying the following conditions:

- (i) $E = E_1 \cap E_2 \neq \emptyset$ and
- (ii) $\tilde{H}[e] = \tilde{F}[e] \cup \tilde{G}[e]$ for all $e \in E$.

Definition 4.4.3 Let (\tilde{F}, E_1) and (\tilde{G}, E_2) be two fuzzy soft sets over a common universe U . The *extended intersection* [28] of (\tilde{F}, E_1) and (\tilde{G}, E_2) is defined to be the fuzzy soft set $(\tilde{F}, E_1) \cap (\tilde{G}, E_2) = (\tilde{H}, E)$ satisfying the following conditions:

(i) $E = E_1 \cup E_2$ and

(ii) for all $e \in E$,

$$\tilde{H}[e] = \begin{cases} \tilde{F}[e] & \text{if } e \in E_1 \setminus E_2 \\ \tilde{G}[e] & \text{if } e \in E_2 \setminus E_1 \\ \tilde{F}[e] \cap \tilde{G}[e] & \text{if } e \in E_1 \cap E_2. \end{cases}$$

The *intersection* [2] of (\tilde{F}, E_1) and (\tilde{G}, E_2) is defined to be the fuzzy soft set (\tilde{H}, E) satisfying the following conditions:

(i) $E = E_1 \cap E_2 \neq \emptyset$ and

(ii) $\tilde{H}[e] = \tilde{F}[e] \cap \tilde{G}[e]$ for all $e \in E$.

Definition 4.4.4 A fuzzy soft set (\tilde{F}, E) over A is called a *fuzzy soft UP_s -subalgebra* based on $e \in E$ (we shortly call an *e-fuzzy soft UP_s -subalgebra*) of A if a fuzzy set $\tilde{F}[e]$ in A is a fuzzy UP_s -subalgebra of A . If (\tilde{F}, E) is an *e-fuzzy soft UP_s -subalgebra* of A for all $e \in E$, we say that (\tilde{F}, E) is a *fuzzy soft UP_s -subalgebra* of A .

In the next theorem, we give necessary condition for fuzzy soft UP_s -subalgebras of f -UP-semigroups.

Theorem 4.4.5 *If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.3) and (3.0.14), then (\tilde{F}, E) is a fuzzy soft UP_s -subalgebra of A .*

Proof. It is straightforward by Proposition 4.3.3 and Lemma 3.0.36 (1). \square

The proof of the following theorem can be verified easily.

Theorem 4.4.6 *If (\tilde{F}, E) is a fuzzy soft UP_s -subalgebra of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft UP_s -subalgebra of A .*

The following example shows that there exists a nonempty subset E^* of E such that $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft UP_s -subalgebra of A , but (\tilde{F}, E) is not a fuzzy soft UP_s -subalgebra of A .

Example 4.4.7 Let A be the set of four series of the iPhone, that is,

$$A = \{5, 6, 7, X\}.$$

Define two binary operations \cdot and $*$ on A as the following Cayley tables:

\cdot	X	7	6	5
X	X	7	6	5
7	X	X	6	5
6	X	7	X	5
5	X	7	6	X

$*$	X	7	6	5
X	X	X	X	X
7	X	X	X	X
6	X	X	X	7
5	X	X	7	X

Then $A = (A, \cdot, *, X)$ is an f -UP-semigroup. Let (\tilde{F}, E) be a fuzzy soft set over A where

$$E := \{\text{price, beauty, specifications, stability}\}$$

with $\tilde{F}[\text{price}]$, $\tilde{F}[\text{beauty}]$, $\tilde{F}[\text{specifications}]$, and $\tilde{F}[\text{stability}]$ are fuzzy sets in A defined as follows:

\tilde{F}	X	7	6	5
price	0.8	0.3	0.7	0.1
beauty	0.5	0.3	0.2	0.4
specifications	0.9	0.8	0.5	0.6
stability	1	0.4	0.7	0.6

Then $\tilde{F}[\text{stability}]$ is not a fuzzy UP_s -subalgebra of A . Indeed,

$$\begin{aligned} f_{\tilde{F}[\text{stability}]}(5 * 6) &= f_{\tilde{F}[\text{stability}]}(7) = 0.4 \not\geq 0.6 = \min\{0.6, 0.7\} = \\ &\min\{f_{\tilde{F}[\text{stability}]}(5), f_{\tilde{F}[\text{stability}]}(6)\}. \end{aligned}$$

Hence, (\tilde{F}, E) is not a fuzzy soft UP_s -subalgebra of A . We take

$$E^* := \{\text{price, beauty, specifications}\}.$$

Thus $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft UP_s -subalgebra of A .

Theorem 4.4.8 *The extended intersection of two fuzzy soft UP_s -subalgebras of A is also a fuzzy soft UP_s -subalgebra. Moreover, the intersection of two fuzzy soft UP_s -subalgebras of A is also a fuzzy soft UP_s -subalgebra.*

Proof. Let (\tilde{F}, E_1) and (\tilde{G}, E_2) be two fuzzy soft UP_s -subalgebras of A . Assume that $(\tilde{F}, E_1) \cap (\tilde{G}, E_2) = (\tilde{H}, E)$ with $E = E_1 \cup E_2$. Let $e \in E$.

Case 1: $e \in E_1 \setminus E_2$ (resp., $e \in E_2 \setminus E_1$). Then $\tilde{H}[e] = \tilde{F}[e]$ (resp., $\tilde{H}[e] = \tilde{G}[e]$) is a fuzzy soft UP_s -subalgebra of A .

Case 2: $e \in E_1 \cap E_2$. By Theorem 4.2.2, we have $\tilde{H}[e] = \tilde{F}[e] \cap \tilde{G}[e]$ is a fuzzy soft UP_s -subalgebra.

Thus (\tilde{H}, E) is an e -fuzzy soft UP_s -subalgebra of A for all $e \in E$. Hence, (\tilde{H}, E) is a fuzzy soft UP_s -subalgebra of A . \square

Theorem 4.4.9 *The union of two fuzzy soft UP_s -subalgebras of A is also a fuzzy soft UP_s -subalgebra if sets of statistics of two fuzzy soft UP_s -subalgebras are disjoint.*

Proof. Let (\tilde{F}, E_1) and (\tilde{G}, E_2) be two fuzzy soft UP_s -subalgebras of A such that $E_1 \cap E_2 = \emptyset$. Assume that $(\tilde{F}, E_1) \cup (\tilde{G}, E_2) = (\tilde{H}, E)$ with $E = E_1 \cup E_2$. Let $e \in E$. Since $E_1 \cap E_2 = \emptyset$, we have $e \in E_1 \setminus E_2$ or $e \in E_2 \setminus E_1$.

Case 1: $e \in E_1 \setminus E_2$. Then $\tilde{H}[e] = \tilde{F}[e]$ is a fuzzy soft UP_s -subalgebra of A .

Case 2: $e \in E_2 \setminus E_1$. Then $\tilde{H}[e] = \tilde{G}[e]$ is a fuzzy soft UP_s -subalgebra of A .

Thus (\tilde{H}, E) is an e -fuzzy soft UP_s -subalgebra of A for all $e \in E$. Hence, (\tilde{H}, E) is a fuzzy soft UP_s -subalgebra of A . \square

The following example shows that Theorem 4.4.9 is not valid if sets of statistics of two fuzzy soft UP_s -subalgebras are not disjoint.

Example 4.4.10 By Cayley tables in Example 4.4.7, we know that $A = (A, \cdot, *, X)$ is an f -UP-semigroup. Let (\tilde{G}_1, E_1) and (\tilde{G}_2, E_2) be two fuzzy soft sets over A where

$$E_1 := \{\text{price, beauty, specifications}\} \text{ and } E_2 := \{\text{price, stability}\}$$

with $\tilde{G}_1[\text{price}]$, $\tilde{G}_1[\text{beauty}]$, $\tilde{G}_1[\text{specifications}]$, $\tilde{G}_2[\text{price}]$, and $\tilde{G}_2[\text{stability}]$ are fuzzy sets in A defined as follows:

\tilde{G}_1	X	7	6	5
price	0.9	0.7	0.9	0.2
beauty	1	0.8	0.3	0.2
specifications	0.6	0.5	0.3	0.4
\tilde{G}_2	X	7	6	5
price	0.9	0.3	0.2	0.8
stability	0.7	0.2	0.5	0.2

Then (\tilde{G}_1, E_1) and (\tilde{G}_2, E_2) are two fuzzy soft UP_s -subalgebras of A . Since $\text{price} \in E_1 \cap E_2$, we have

$$(f_{\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]})(6 * 5) = (f_{\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]})(7)$$

$$\begin{aligned}
&= 0.7 \\
&\neq 0.8 \\
&= \min\{0.9, 0.8\} \\
&= \min\{(f_{\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]})^{(6)}, (f_{\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]})^{(5)}\}.
\end{aligned}$$

Thus $\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]$ is not a fuzzy UP_s -subalgebra of A , that is, $(\tilde{G}_1, E_1) \cup (\tilde{G}_2, E_2)$ is not a price-fuzzy soft UP_s -subalgebra of A . Hence, $(\tilde{G}_1, E_1) \cup (\tilde{G}_2, E_2)$ is not a fuzzy soft UP_s -subalgebra of A . Moreover, $(\tilde{G}_1, E_1) \cup (\tilde{G}_2, E_2)$ is not a fuzzy soft UP_s -subalgebra of A .

Definition 4.4.11 A fuzzy soft set (\tilde{F}, E) over A is called a *fuzzy soft UP_i -subalgebra* based on $e \in E$ (we shortly call an *e-fuzzy soft UP_i -subalgebra*) of A if a fuzzy set $\tilde{F}[e]$ in A is a fuzzy UP_i -subalgebra of A . If (\tilde{F}, E) is an *e-fuzzy soft UP_i -subalgebra* of A for all $e \in E$, we say that (\tilde{F}, E) is a *fuzzy soft UP_i -subalgebra* of A .

In the next theorem, we give necessary condition for fuzzy soft UP_i -subalgebras of f -UP-semigroups.

Theorem 4.4.12 *If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.3) and (3.0.15), then (\tilde{F}, E) is a fuzzy soft UP_i -subalgebra of A .*

Proof. It is straightforward by Proposition 4.3.3 and Lemma 3.0.36 (2). \square

Theorem 4.4.13 *Every e-fuzzy soft UP_i -subalgebra of A is an e-fuzzy soft UP_s -subalgebra. Moreover, every fuzzy soft UP_i -subalgebra of A is a fuzzy soft UP_s -subalgebra.*

The following example shows that the converse of Theorem 4.4.13 is not true.

Example 4.4.14 In Example 4.4.7, we know that (\tilde{F}, E) is a price-fuzzy soft UP_s -subalgebra of A but $\tilde{F}[\text{price}]$ is not a fuzzy UP_i -subalgebra of A . Indeed,

$$f_{\tilde{F}[\text{price}]}(6 * 5) = f_{\tilde{F}[\text{price}]}(7) = 0.3 \not\geq 0.7 = \max\{0.7, 0.1\} = \max\{f_{\tilde{F}[\text{price}]}(6), f_{\tilde{F}[\text{price}]}(5)\}.$$

Hence, (\tilde{F}, E) is not a price-fuzzy soft UP_i -subalgebra of A .

The proof of the following theorem can be verified easily.

Theorem 4.4.15 *If (\tilde{F}, E) is a fuzzy soft UP_i -subalgebra of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft UP_i -subalgebra of A .*

The following two theorems can be deduced in the same way as Theorems 4.4.8 and 4.4.9.

Theorem 4.4.16 *The extended intersection of two fuzzy soft UP_i -subalgebras of A is also a fuzzy soft UP_i -subalgebra. Moreover, the intersection of two fuzzy soft UP_i -subalgebras of A is also a fuzzy soft UP_i -subalgebra.*

Theorem 4.4.17 *The union of two fuzzy soft UP_i -subalgebras of A is also a fuzzy soft UP_i -subalgebra if sets of statistics of two fuzzy soft UP_i -subalgebras are disjoint.*

The following example shows that Theorem 4.4.17 is not valid if sets of statistics of two fuzzy soft UP_i -subalgebras are not disjoint.

Example 4.4.18 Let A be the set of four types of a music, that is,

$$A = \{\text{pop, rock, classic, disco}\}.$$

Define two binary operations \cdot and $*$ on A as the following Cayley tables:

\cdot	pop	rock	disco	classic
pop	pop	rock	disco	classic
rock	pop	pop	disco	disco
disco	pop	rock	pop	disco
classic	pop	rock	pop	pop
$*$	pop	rock	disco	classic
pop	pop	pop	pop	pop
rock	pop	pop	pop	pop
disco	pop	pop	pop	pop
classic	pop	pop	pop	pop

Then $A = (A, \cdot, *, \text{pop})$ is an f -UP-semigroup. Let (\tilde{G}_1, E_1) and (\tilde{G}_2, E_2) be two fuzzy soft sets over A where

$$E_1 := \{\text{sorrow, modernity}\} \text{ and } E_2 := \{\text{modernity, enjoyment}\}$$

with $\tilde{G}_1[\text{sorrow}]$, $\tilde{G}_1[\text{modernity}]$, $\tilde{G}_2[\text{modernity}]$, and $\tilde{G}_2[\text{enjoyment}]$ are fuzzy sets in A defined as follows:

\tilde{G}_1	pop	rock	disco	classic
sorrow	0.7	0.7	0.5	0.5
modernity	0.9	0.8	0.3	0.3

\tilde{G}_2	pop	rock	disco	classic
modernity	0.8	0.3	0.4	0.5
enjoyment	1	0.9	0.1	0.1

Then (\tilde{G}_1, E_1) and (\tilde{G}_2, E_2) are two fuzzy soft UP_i-subalgebras of A . Since

modernity $\in E_1 \cap E_2$, we have

$$\begin{aligned}
& (f_{\tilde{G}_1[\text{modernity}] \cup \tilde{G}_2[\text{modernity}]})(\text{rock} \cdot \text{classic}) \\
&= (f_{\tilde{G}_1[\text{modernity}] \cup \tilde{G}_2[\text{modernity}]})(\text{disco}) \\
&= 0.4 \\
&\not\geq 0.5 \\
&= \min\{0.8, 0.5\} \\
&= \min\{(f_{\tilde{G}_1[\text{modernity}] \cup \tilde{G}_2[\text{modernity}]})(\text{rock}), (f_{\tilde{G}_1[\text{modernity}] \cup \tilde{G}_2[\text{modernity}]})(\text{classic})\}.
\end{aligned}$$

Thus $\tilde{G}_1[\text{modernity}] \cup \tilde{G}_2[\text{modernity}]$ is not a fuzzy UP_i -subalgebra of A , that is, $(\tilde{G}_1, E_1) \cup (\tilde{G}_2, E_2)$ is not a modernity-fuzzy soft UP_i -subalgebra of A . Hence, $(\tilde{G}_1, E_1) \cup (\tilde{G}_2, E_2)$ is not a fuzzy soft UP_i -subalgebra of A . Moreover, $(\tilde{G}_1, E_1) \cup (\tilde{G}_2, E_2)$ is not a fuzzy soft UP_i -subalgebra of A .

Definition 4.4.19 A fuzzy soft set (\tilde{F}, E) over A is called a *fuzzy soft near UP_s -filter* based on $e \in E$ (we shortly call an *e -fuzzy soft near UP_s -filter*) of A if a fuzzy set $\tilde{F}[e]$ in A is a fuzzy near UP_s -filter of A . If (\tilde{F}, E) is an e -fuzzy soft near UP_s -filter of A for all $e \in E$, we say that (\tilde{F}, E) is a *fuzzy soft near UP_s -filter* of A .

In the next theorem, we give necessary condition for fuzzy soft near UP_s -filters of f - UP -semigroups.

Theorem 4.4.20 *If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.2) and (3.0.14), then (\tilde{F}, E) is a fuzzy soft near UP_s -filter of A .*

Proof. It is straightforward by Proposition 4.3.8 and Lemma 3.0.36 (1). \square

Theorem 4.4.21 *Every e -fuzzy soft near UP_s -filter of A is an e -fuzzy soft UP_s -*

subalgebra. Moreover, every fuzzy soft near UP_s -filter of A is a fuzzy soft UP_s -subalgebra.

The following example shows that the converse of Theorem 4.4.21 is not true.

Example 4.4.22 Let A be a set of four foods, that is,

$$A = \{\text{apple, banana, meat, rice}\}.$$

Define two binary operations \cdot and $*$ on A as the following Cayley tables:

\cdot	rice	apple	banana	meat
rice	rice	apple	banana	meat
apple	rice	rice	apple	meat
banana	rice	rice	rice	meat
meat	rice	apple	apple	rice

$*$	rice	apple	banana	meat
rice	rice	rice	rice	rice
apple	rice	rice	rice	rice
banana	rice	rice	rice	rice
meat	rice	rice	rice	apple

Then $A = (A, \cdot, *, \text{rice})$ is an f -UP-semigroup. Let (\tilde{F}, E) be a fuzzy soft set over A where

$$E := \{\text{pig, monkey, chicken}\}$$

with $\tilde{F}[\text{pig}]$, $\tilde{F}[\text{monkey}]$, and $\tilde{F}[\text{chicken}]$ are fuzzy sets in A defined as follows:

\tilde{F}	rice	apple	banana	meat
pig	1	0.8	0.9	0.3
monkey	0.8	0.4	0.8	0.3
chicken	0.7	0.4	0.3	0.2

Then (\tilde{F}, E) is a pig-fuzzy soft UP_s -subalgebra of A . But (\tilde{F}, E) is not a pig-fuzzy soft near UP_s -filter of A since

$$\begin{aligned} f_{\tilde{F}[\text{pig}]}(\text{meat} \cdot \text{banana}) &= f_{\tilde{F}[\text{pig}]}(\text{apple}) \\ &= 0.8 \\ &\not\geq 0.9 \\ &= f_{\tilde{F}[\text{pig}]}(\text{banana}), \end{aligned}$$

that is, $\tilde{F}[\text{pig}]$ is not a fuzzy near UP_s -filter of A .

In the next theorem, we give necessary condition for fuzzy soft UP_s -subalgebras as fuzzy soft near UP_s -filters of f - UP -semigroups.

Theorem 4.4.23 *If (\tilde{F}, E) is a fuzzy soft UP_s -subalgebra of A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the condition (4.3.5), then (\tilde{F}, E) is a fuzzy soft near UP_s -filter of A .*

Proof. It is straightforward by Theorem 4.3.11. □

The proof of the following theorem can be verified easily.

Theorem 4.4.24 *If (\tilde{F}, E) is a fuzzy soft near UP_s -filter of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft near UP_s -filter of A .*

The following two theorems can be deduced in the same way as Theorems 4.4.8 and 4.4.9.

Theorem 4.4.25 *The extended intersection of two fuzzy soft near UP_s -filters of A is also a fuzzy soft near UP_s -filter. Moreover, the intersection of two fuzzy soft near UP_s -filters of A is also a fuzzy soft near UP_s -filter.*

Theorem 4.4.26 *The union of two fuzzy soft near UP_s -filters of A is also a fuzzy soft near UP_s -filter if sets of statistics of two fuzzy soft near UP_s -filters are disjoint.*

The following example shows that Theorem 4.4.26 is not valid if sets of statistics of two fuzzy soft near UP_s -filters are not disjoint.

Example 4.4.27 In Example 4.4.10, we have (\tilde{G}_1, E_1) and (\tilde{G}_2, E_2) are two fuzzy soft near UP_s -filters of A . Since price $\in E_1 \cap E_2$, we have

$$\begin{aligned} (f_{\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]})(6 * 5) &= (f_{\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]})(7) \\ &= 0.7 \\ &\not\geq 0.8 \\ &= \min\{0.9, 0.8\} \\ &= \min\{(f_{\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]})(6), (f_{\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]})(5)\}. \end{aligned}$$

Thus $\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]$ is not a fuzzy near UP_s -filter of A , that is, $(\tilde{G}_1, E_1) \cup (\tilde{G}_2, E_2)$ is not a price-fuzzy soft near UP_s -filter of A . Hence, $(\tilde{G}_1, E_1) \cup (\tilde{G}_2, E_2)$ is not a fuzzy soft near UP_s -filter of A . Moreover, $(\tilde{G}_1, E_1) \uplus (\tilde{G}_2, E_2)$ is not a fuzzy soft near UP_s -filter of A .

Definition 4.4.28 A fuzzy soft set (\tilde{F}, E) over A is called a *fuzzy soft near UP_1 -filter* based on $e \in E$ (we shortly call an *e-fuzzy soft near UP_1 -filter*) of A if a fuzzy set $\tilde{F}[e]$ in A is a fuzzy near UP_1 -filter of A . If (\tilde{F}, E) is an *e-fuzzy soft near*

UP_i -filter of A for all $e \in E$, we say that (\tilde{F}, E) is a *fuzzy soft near UP_i -filter* of A .

In the next theorem, we give necessary condition for fuzzy soft near UP_i -filters of f -UP-semigroups.

Theorem 4.4.29 *If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.2) and (3.0.15), then (\tilde{F}, E) is a fuzzy soft near UP_i -filter of A .*

Proof. It is straightforward by Proposition 4.3.8 and Lemma 3.0.36 (2). \square

Theorem 4.4.30 *Every e -fuzzy soft near UP_i -filter of A is an e -fuzzy soft near UP_s -filter. Moreover, every fuzzy soft near UP_i -filter of A is a fuzzy soft near UP_s -filter.*

Theorem 4.4.31 *Every e -fuzzy soft near UP_i -filter of A is an e -fuzzy soft UP_i -subalgebra. Moreover, every fuzzy soft near UP_i -filter of A is a fuzzy soft UP_i -subalgebra.*

The following two examples show that the converse of Theorems 4.4.30 and 4.4.31 is not true.

Example 4.4.32 In Example 4.4.7, we know that (\tilde{F}, E) is a price-fuzzy soft near UP_s -filter of A but $\tilde{F}[\text{price}]$ is not a fuzzy near UP_i -filter of A . Indeed,

$$f_{\tilde{F}[\text{price}]}(6 * 5) = f_{\tilde{F}[\text{price}]}(7) = 0.3 \not\geq 0.7 = \max\{0.7, 0.1\} = \max\{f_{\tilde{F}[\text{price}]}(6), f_{\tilde{F}[\text{price}]}(5)\}.$$

Hence, (\tilde{F}, E) is not a price-fuzzy soft near UP_i -filter of A .

Example 4.4.33 In Example 4.4.22, we know that (\tilde{F}, E) is a monkey-fuzzy soft UP_i -subalgebra of A but $\tilde{F}[\text{monkey}]$ is not a fuzzy near UP_i -filter of A . Indeed,

$$f_{\tilde{F}[\text{monkey}]}(\text{apple} \cdot \text{banana}) = f_{\tilde{F}[\text{monkey}]}(\text{apple}) = 0.4 \not\geq 0.8 = f_{\tilde{F}[\text{monkey}]}(\text{banana}).$$

Hence, (\tilde{F}, E) is not a monkey-fuzzy soft near UP_i -filter of A .

In the next theorem, we give necessary condition for fuzzy soft UP_i -subalgebras as fuzzy soft near UP_i -filters of f -UP-semigroups.

Theorem 4.4.34 *If (\tilde{F}, E) is a fuzzy soft UP_i -subalgebra of A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the condition (4.3.5), then (\tilde{F}, E) is a fuzzy soft near UP_i -filter of A .*

Proof. It is straightforward by Theorem 4.3.11. □

The proof of the following theorem can be verified easily.

Theorem 4.4.35 *If (\tilde{F}, E) is a fuzzy soft near UP_i -filter of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft near UP_i -filter of A .*

By using Theorem 4.2.10, we can obtain the following two theorems in the same way as Theorems 4.4.8 and 4.4.9.

Theorem 4.4.36 *The extended intersection of two fuzzy soft near UP_i -filters of A is also a fuzzy soft near UP_i -filter. Moreover, the intersection of two fuzzy soft near UP_i -filters of A is also a fuzzy soft near UP_i -filter.*

Theorem 4.4.37 *The union of two fuzzy soft near UP_i -filters of A is also a fuzzy soft near UP_i -filter. Moreover, the restricted union of two fuzzy soft near UP_i -filters of A is also a fuzzy soft near UP_i -filter.*

Definition 4.4.38 A fuzzy soft set (\tilde{F}, E) over A is called a *fuzzy soft UP_s -filter* based on $e \in E$ (we shortly call an *e-fuzzy soft UP_s -filter*) of A if a fuzzy set $\tilde{F}[e]$ in A is a fuzzy UP_s -filter of A . If (\tilde{F}, E) is an *e-fuzzy soft UP_s -filter* of A for all $e \in E$, we say that (\tilde{F}, E) is a *fuzzy soft UP_s -filter* of A .

In the next theorem, we give necessary condition for fuzzy soft UP_s -filters of f -UP-semigroups.

Theorem 4.4.39 *If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.6) and (3.0.14), then (\tilde{F}, E) is a fuzzy soft UP_s -filter of A .*

Proof. It is straightforward by Proposition 4.3.12 and Lemma 3.0.36 (1). \square

Theorem 4.4.40 *Every e -fuzzy soft UP_s -filter of A is an e -fuzzy soft near UP_s -filter. Moreover, every fuzzy soft UP_s -filter of A is a fuzzy soft near UP_s -filter.*

The following example shows that the converse of Theorem 4.4.40 is not true.

Example 4.4.41 Let A be a set of four coffees, that is,

$$A = \{\text{Mocha}(M), \text{Americano}(A), \text{Cappuccino}(C), \text{Latte}(L)\}.$$

Define two binary operations \cdot and $*$ on A as the following Cayley tables:

\cdot	L	A	M	C
L	L	A	M	C
A	L	L	M	C
M	L	L	L	C
C	L	L	L	L

\cdot	L	A	M	C
L	L	L	L	L
A	L	L	L	L
M	L	L	L	L
C	L	L	L	M

Then $A = (A, \cdot, *, \text{Latte})$ is an f -UP-semigroup. Let (\tilde{F}, E) be a fuzzy soft set over A where

$$E := \{\text{sweetness, strong, aroma}\}$$

with $\tilde{F}[\text{sweetness}]$, $\tilde{F}[\text{strong}]$, and $\tilde{F}[\text{aroma}]$ are fuzzy sets in A defined as follows:

\tilde{F}	L	A	M	C
sweetness	0.8	0.1	0.6	0.6
strong	0.7	0.7	0.6	0.5
aroma	0.5	0.3	0.4	0.1

Then (\tilde{F}, E) is a sweetness-fuzzy soft near UP_s -filter of A but $\tilde{F}[\text{sweetness}]$ is not a fuzzy UP_s -filter of A . Indeed,

$$\begin{aligned} f_{\tilde{F}[\text{sweetness}]}(A) &= 0.1 \not\geq 0.6 = \min\{0.8, 0.6\} = \\ \min\{f_{\tilde{F}[\text{sweetness}]}(L), f_{\tilde{F}[\text{sweetness}]}(M)\} &= \min\{f_{\tilde{F}[\text{sweetness}]}(M \cdot A), f_{\tilde{F}[\text{sweetness}]}(M)\} \end{aligned}$$

Hence, (\tilde{F}, E) is not a sweetness-fuzzy soft UP_s -filter of A .

In the next theorem, we give necessary condition for fuzzy soft near UP_s -filters as fuzzy soft UP_s -filters of f - UP -semigroups.

Theorem 4.4.42 *If (\tilde{F}, E) is a fuzzy soft near UP_s -filter of A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the condition (4.3.7), then (\tilde{F}, E) is a fuzzy soft UP_s -filter of A .*

Proof. It is straightforward by Theorem 4.3.15. □

The proof of the following theorem can be verified easily.

Theorem 4.4.43 *If (\tilde{F}, E) is a fuzzy soft UP_s -filter of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft UP_s -filter of A .*

The following two theorems can be deduced in the same way as Theorems 4.4.8 and 4.4.9.

Theorem 4.4.44 *The extended intersection of two fuzzy soft UP_s -filters of A is also a fuzzy soft UP_s -filter. Moreover, the intersection of two fuzzy soft UP_s -filters of A is also a fuzzy soft UP_s -filter.*

Theorem 4.4.45 *The union of two fuzzy soft UP_s -filters of A is also a fuzzy soft UP_s -filter if sets of statistics of two fuzzy soft UP_s -filters are disjoint.*

The following example shows that Theorem 4.4.45 is not valid if sets of statistics of two fuzzy soft UP_s -filters are not disjoint.

Example 4.4.46 In Example 4.4.10, we have (\tilde{G}_1, E_1) and (\tilde{G}_2, E_2) are two fuzzy soft UP_s -filters of A . Since $\text{price} \in E_1 \cap E_2$, we have

$$\begin{aligned} (f_{\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]})(6 * 5) &= (f_{\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]})(7) = 0.7 \not\geq 0.8 = \min\{0.9, 0.8\} = \\ &= \min\{(f_{\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]})(6), (f_{\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]})(5)\}. \end{aligned}$$

Thus $\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]$ is not a fuzzy UP_s -filter of A , that is, $(\tilde{G}_1, E_1) \cup (\tilde{G}_2, E_2)$ is not a price-fuzzy soft UP_s -filter of A . Hence, $(\tilde{G}_1, E_1) \cup (\tilde{G}_2, E_2)$ is not a fuzzy soft UP_s -filter of A . Moreover, $(\tilde{G}_1, E_1) \uplus (\tilde{G}_2, E_2)$ is not a fuzzy soft UP_s -filter of A .

Definition 4.4.47 A fuzzy soft set (\tilde{F}, E) over A is called a *fuzzy soft UP_i -filter* based on $e \in E$ (we shortly call an *e-fuzzy soft UP_i -filter*) of A if a fuzzy set $\tilde{F}[e]$ in A is a fuzzy UP_i -filter of A . If (\tilde{F}, E) is an *e-fuzzy soft UP_i -filter* of A for all $e \in E$, we say that (\tilde{F}, E) is a *fuzzy soft UP_i -filter* of A .

In the next theorem, we give necessary condition for fuzzy soft UP_i -filters of f -UP-semigroups.

Theorem 4.4.48 *If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.6) and (3.0.15), then (\tilde{F}, E) is a fuzzy soft UP_i -filter of A .*

Proof. It is straightforward by Proposition 4.3.12 and Lemma 3.0.36 (2). \square

Theorem 4.4.49 *Every e -fuzzy soft UP_i -filter of A is an e -fuzzy soft UP_s -filter.*

Moreover, every fuzzy soft UP_i -filter of A is a fuzzy soft UP_s -filter.

Theorem 4.4.50 *Every e -fuzzy soft UP_i -filter of A is an e -fuzzy soft near UP_i -filter. Moreover, every fuzzy soft UP_i -filter of A is a fuzzy soft near UP_i -filter.*

The following two examples show that the converse of Theorems 4.4.49 and 4.4.50 is not true.

Example 4.4.51 In Example 4.4.7, we know that (\tilde{F}, E) is a beauty-fuzzy soft UP_s -filter of A but $\tilde{F}[\text{beauty}]$ is not a fuzzy UP_i -filter of A . Indeed,

$$f_{\tilde{F}[\text{beauty}]}(6 * 5) = f_{\tilde{F}[\text{beauty}]}(7) = 0.3 \not\geq 0.4 = \max\{0.2, 0.4\} = \max\{f_{\tilde{F}[\text{beauty}]}(6), f_{\tilde{F}[\text{beauty}]}(5)\}.$$

Hence, (\tilde{F}, E) is not a beauty-fuzzy soft UP_i -filter of A .

Example 4.4.52 In Example 4.4.41, we know that (\tilde{F}, E) is a aroma-fuzzy soft near UP_i -filter of A but $\tilde{F}[\text{aroma}]$ is not a fuzzy UP_i -filter of A . Indeed,

$$f_{\tilde{F}[\text{aroma}]}(A) = 0.3 \not\geq 0.4 = \min\{0.5, 0.4\} = \min\{f_{\tilde{F}[\text{aroma}]}(L), f_{\tilde{F}[\text{aroma}]}(M)\} = \min\{f_{\tilde{F}[\text{aroma}]}(M \cdot A), f_{\tilde{F}[\text{aroma}]}(M)\}.$$

Hence, (\tilde{F}, E) is not a aroma-fuzzy soft UP_i -filter of A .

In the next theorem, we give necessary condition for fuzzy soft near UP_i -filters as fuzzy soft UP_i -filters of f -UP-semigroups.

Theorem 4.4.53 *If (\tilde{F}, E) is a fuzzy soft near UP_i -filter of A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the condition (4.3.7), then (\tilde{F}, E) is a fuzzy soft UP_i -filter of A .*

Proof. It is straightforward by Theorem 4.3.15. \square

The proof of the following theorem can be verified easily.

Theorem 4.4.54 *If (\tilde{F}, E) is a fuzzy soft UP_i -filter of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft UP_i -filter of A .*

The following two theorems can be deduced in the same way as Theorems 4.4.8 and 4.4.9.

Theorem 4.4.55 *The extended intersection of two fuzzy soft UP_i -filters of A is also a fuzzy soft UP_i -filter. Moreover, the intersection of two fuzzy soft UP_i -filters of A is also a fuzzy soft UP_i -filter.*

Theorem 4.4.56 *The union of two fuzzy soft UP_i -filters of A is also a fuzzy soft UP_i -filter if sets of statistics of two fuzzy soft UP_i -filters are disjoint.*

The following example shows that Theorem 4.4.56 is not valid if sets of statistics of two fuzzy soft UP_i -filters are not disjoint.

Example 4.4.57 Let A be a set of four colors, that is,

$$A = \{\text{blue, green, cyan, black}\}.$$

Define two binary operations \cdot and $*$ on A as the following Cayley tables:

\cdot	black	cyan	blue	green
black	black	cyan	blue	green
cyan	black	black	blue	blue
blue	black	cyan	black	cyan
green	black	black	black	black

*	black	cyan	blue	green
black	black	black	black	black
cyan	black	black	black	black
blue	black	black	black	black
green	black	black	black	black

Then $A = (A, \cdot, *, \text{black})$ is an f -UP-semigroup. Let (\tilde{G}_1, E_1) and (\tilde{G}_2, E_2) be two fuzzy soft sets over A where

$$E_1 := \{\text{endurance, beauty}\} \text{ and } E_2 := \{\text{endurance, warmth}\}$$

with $\tilde{G}_1[\text{endurance}]$, $\tilde{G}_1[\text{beauty}]$, $\tilde{G}_2[\text{endurance}]$, and $\tilde{G}_2[\text{warmth}]$ are fuzzy sets in A defined as follows:

\tilde{G}_1	black	cyan	blue	green
endurance	1	0.5	0.7	0.5
beauty	0.4	0.3	0.2	0.2

\tilde{G}_2	black	cyan	blue	green
endurance	1	0.6	0.5	0.5
warmth	0.9	0.4	0.5	0.4

Then (\tilde{G}_1, E_1) and (\tilde{G}_2, E_2) are two fuzzy soft UP₁-filters of A . Since $\text{endurance} \in E_1 \cap E_2$, we have

$$\begin{aligned} & (f_{\tilde{G}_1[\text{endurance}] \cup \tilde{G}_2[\text{endurance}]})(\text{green}) = 0.5 \not\geq 0.6 = \min\{0.6, 0.7\} = \\ & \min\{(f_{\tilde{G}_1[\text{endurance}] \cup \tilde{G}_2[\text{endurance}]})(\text{cyan}), (f_{\tilde{G}_1[\text{endurance}] \cup \tilde{G}_2[\text{endurance}]})(\text{blue})\} = \\ & \min\{(f_{\tilde{G}_1[\text{endurance}] \cup \tilde{G}_2[\text{endurance}]})(\text{blue} \cdot \text{green}), (f_{\tilde{G}_1[\text{endurance}] \cup \tilde{G}_2[\text{endurance}]})(\text{blue})\}. \end{aligned}$$

Thus $\tilde{G}_1[\text{endurance}] \cup \tilde{G}_2[\text{endurance}]$ is not a fuzzy UP₁-filter of A , that is, $(\tilde{G}_1, E_1) \cup (\tilde{G}_2, E_2)$ is not an endurance-fuzzy soft UP₁-filter of A . Hence, $(\tilde{G}_1, E_1) \cup$

(\tilde{G}_2, E_2) is not a fuzzy soft UP_i -filter of A . Moreover, $(\tilde{G}_1, E_1) \cup (\tilde{G}_2, E_2)$ is not a fuzzy soft UP_i -filter of A .

Definition 4.4.58 A fuzzy soft set (\tilde{F}, E) over A is called a *fuzzy soft UP_s -ideal* based on $e \in E$ (we shortly call an *e-fuzzy soft UP_s -ideal*) of A if a fuzzy set $\tilde{F}[e]$ in A is a fuzzy UP_s -ideal of A . If (\tilde{F}, E) is an *e-fuzzy soft UP_s -ideal* of A for all $e \in E$, we say that (\tilde{F}, E) is a *fuzzy soft UP_s -ideal* of A .

In the next theorem and corollary, we give necessary condition for fuzzy soft UP_s -ideals of f -UP-semigroups.

Theorem 4.4.59 *If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.8) and (3.0.14), then (\tilde{F}, E) is a fuzzy soft UP_s -ideal of A .*

Proof. It is straightforward by Proposition 4.3.16 and Lemma 3.0.36 (1). \square

Corollary 4.4.60 *Let A be an f -UP-semigroup satisfying the condition (4.3.10). If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.9) and (3.0.14), then (\tilde{F}, E) is a fuzzy soft UP_s -ideal of A .*

Proof. It is straightforward by Theorems 4.4.59 and 4.3.19. \square

Theorem 4.4.61 *Every e-fuzzy soft UP_s -ideal of A is an e-fuzzy soft UP_s -filter. Moreover, every fuzzy soft UP_s -ideal of A is a fuzzy soft UP_s -filter.*

The following example shows that the converse of Theorem 4.4.61 is not true.

Example 4.4.62 By Cayley tables in Example 4.4.18, we know that $A = (A, \cdot, *, \text{pop})$ is an f -UP-semigroup. Let (\tilde{F}, E) be a fuzzy soft set over A where

$$E := \{\text{sorrow, relaxation, enjoyment}\}$$

with $\tilde{F}[\text{sorrow}]$, $\tilde{F}[\text{modernity}]$, and $\tilde{F}[\text{enjoyment}]$ are fuzzy sets in A defined as follows:

\tilde{F}	pop	rock	disco	classic
sorrow	0.6	0.2	0.1	0.1
modernity	1	0.5	0.5	0.5
enjoyment	0.7	0.5	0.2	0.2

Then (\tilde{F}, E) is a sorrow-fuzzy soft UP_s -filter of A but $\tilde{F}[\text{sorrow}]$ is not a fuzzy UP_s -ideal of A . Indeed,

$$\begin{aligned} f_{\tilde{F}[\text{sorrow}]}(\text{disco} \cdot \text{classic}) &= f_{\tilde{F}[\text{sorrow}]}(\text{disco}) = 0.1 \not\geq 0.2 = \min\{0.6, 0.2\} = \\ &= \min\{f_{\tilde{F}[\text{sorrow}]}(\text{pop}), f_{\tilde{F}[\text{sorrow}]}(\text{rock})\} = \\ &= \min\{f_{\tilde{F}[\text{sorrow}]}(\text{disco} \cdot (\text{rock} \cdot \text{classic})), f_{\tilde{F}[\text{sorrow}]}(\text{rock})\}. \end{aligned}$$

Hence, (\tilde{F}, E) is not a sorrow-fuzzy soft UP_s -ideal of A .

In the next theorem, we give necessary condition for fuzzy soft UP_s -filters as fuzzy soft UP_s -ideals of f - UP -semigroups.

Theorem 4.4.63 *If (\tilde{F}, E) is a fuzzy soft UP_s -filter of A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the condition (4.3.11), then (\tilde{F}, E) is a fuzzy soft UP_s -ideal of A .*

Proof. It is straightforward by Theorem 4.3.24. □

The proof of the following theorem can be verified easily.

Theorem 4.4.64 *If (\tilde{F}, E) is a fuzzy soft UP_s -ideal of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft UP_s -ideal of A .*

The following two theorems can be deduced in the same way as Theorems 4.4.8 and 4.4.9.

Theorem 4.4.65 *The extended intersection of two fuzzy soft UP_s -ideals of A is also a fuzzy soft UP_s -ideal. Moreover, the intersection of two fuzzy soft UP_s -ideals of A is also a fuzzy soft UP_s -ideal.*

Theorem 4.4.66 *The union of two fuzzy soft UP_s -ideals of A is also a fuzzy soft UP_s -ideal if sets of statistics of two fuzzy soft UP_s -ideals are disjoint.*

The following example shows that Theorem 4.4.66 is not valid if sets of statistics of two fuzzy soft UP_s -ideals are not disjoint.

Example 4.4.67 In Example 4.4.10, we have (\tilde{G}_1, E_1) and (\tilde{G}_2, E_2) are two fuzzy soft UP_s -ideals of A . Since $\text{price} \in E_1 \cap E_2$, we have

$$\begin{aligned} (f_{\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]})(6 * 5) &= (f_{\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]})(7) = 0.7 \not\geq 0.8 = \min\{0.9, 0.8\} = \\ &= \min\{(f_{\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]})(6), (f_{\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]})(5)\}. \end{aligned}$$

Thus $\tilde{G}_1[\text{price}] \cup \tilde{G}_2[\text{price}]$ is not a fuzzy UP_s -ideal of A , that is, $(\tilde{G}_1, E_1) \cup (\tilde{G}_2, E_2)$ is not a price-fuzzy soft UP_s -ideal of A . Hence, $(\tilde{G}_1, E_1) \cup (\tilde{G}_2, E_2)$ is not a fuzzy soft UP_s -ideal of A . Moreover, $(\tilde{G}_1, E_1) \uplus (\tilde{G}_2, E_2)$ is not a fuzzy soft UP_s -ideal of A .

Definition 4.4.68 A fuzzy soft set (\tilde{F}, E) over A is called a *fuzzy soft UP_i -ideal* based on $e \in E$ (we shortly call an *e-fuzzy soft UP_i -ideal*) of A if a fuzzy set $\tilde{F}[e]$ in A is a fuzzy UP_i -ideal of A . If (\tilde{F}, E) is an *e-fuzzy soft UP_i -ideal* of A for all $e \in E$, we say that (\tilde{F}, E) is a *fuzzy soft UP_i -ideal* of A .

In the next theorem and corollary, we give necessary condition for fuzzy soft UP_i -ideals of f -UP-semigroups.

Theorem 4.4.69 *If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.8) and (3.0.15), then (\tilde{F}, E) is a fuzzy soft UP_i -ideal of A .*

Proof. It is straightforward by Proposition 4.3.16 and Lemma 3.0.36 (2). \square

Corollary 4.4.70 *Let A be an f -UP-semigroup satisfying the condition (4.3.10). If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.9) and (3.0.15), then (\tilde{F}, E) is a fuzzy soft UP_1 -ideal of A .*

Proof. It is straightforward by Theorems 4.4.69 and 4.3.19. \square

Theorem 4.4.71 *Every e -fuzzy soft UP_1 -ideal of A is an e -fuzzy soft UP_s -ideal. Moreover, every fuzzy soft UP_1 -ideal of A is a fuzzy soft UP_s -ideal.*

Theorem 4.4.72 *Every e -fuzzy soft UP_1 -ideal of A is an e -fuzzy soft UP_1 -filter. Moreover, every fuzzy soft UP_1 -ideal of A is a fuzzy soft UP_1 -filter.*

The following two examples show that the converse of Theorems 4.4.71 and 4.4.72 is not true.

Example 4.4.73 In Example 4.4.7, we know that (\tilde{F}, E) is a price-fuzzy soft UP_s -ideal of A but $\tilde{F}[\text{price}]$ is not a fuzzy UP_1 -ideal of A . Indeed,

$$\begin{aligned} f_{\tilde{F}[\text{price}]}(5 * 6) &= f_{\tilde{F}[\text{price}]}(7) = 0.3 \not\geq 0.7 = \max\{0.1, 0.7\} = \\ &= \max\{f_{\tilde{F}[\text{price}]}(5), f_{\tilde{F}[\text{price}]}(6)\}. \end{aligned}$$

Hence, (\tilde{F}, E) is not a price-fuzzy soft UP_1 -ideal of A .

Example 4.4.74 In Example 4.4.62, we know that (\tilde{F}, E) is a enjoyment-fuzzy soft UP_1 -filter of A but $\tilde{F}[\text{enjoyment}]$ is not a fuzzy UP_1 -ideal of A . Indeed,

$$\begin{aligned} f_{\tilde{F}[\text{enjoyment}]}(\text{disco} \cdot \text{classic}) &= f_{\tilde{F}[\text{enjoyment}]}(\text{disco}) = 0.2 \not\geq 0.5 = \min\{0.7, 0.5\} = \\ &= \min\{f_{\tilde{F}[\text{enjoyment}]}(\text{pop}), f_{\tilde{F}[\text{enjoyment}]}(\text{rock})\} = \\ &= \min\{f_{\tilde{F}[\text{enjoyment}]}(\text{disco} \cdot (\text{rock} \cdot \text{classic})), f_{\tilde{F}[\text{enjoyment}]}(\text{rock})\}. \end{aligned}$$

Hence, (\tilde{F}, E) is not a enjoyment-fuzzy soft UP_i -ideal of A .

In the next theorem, we give necessary condition for fuzzy soft UP_i -filters as fuzzy soft UP_i -ideals of f -UP-semigroups.

Theorem 4.4.75 *If (\tilde{F}, E) is a fuzzy soft UP_i -filter of A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the condition (4.3.11), then (\tilde{F}, E) is a fuzzy soft UP_i -ideal of A .*

Proof. It is straightforward by Theorem 4.3.24. □

The proof of the following theorem can be verified easily.

Theorem 4.4.76 *If (\tilde{F}, E) is a fuzzy soft UP_i -ideal of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft UP_i -ideal of A .*

The following two theorems can be deduced in the same way as Theorems 4.4.8 and 4.4.9.

Theorem 4.4.77 *The extended intersection of two fuzzy soft UP_i -ideals of A is also a fuzzy soft UP_i -ideal. Moreover, the intersection of two fuzzy soft UP_i -ideals of A is also a fuzzy soft UP_i -ideal.*

Theorem 4.4.78 *The union of two fuzzy soft UP_i -ideals of A is also a fuzzy soft UP_i -ideal if sets of statistics of two fuzzy soft UP_i -ideals are disjoint.*

The following example shows that the converse of Theorem 4.4.78 is not true.

Example 4.4.79 In Example 4.4.57, we have (\tilde{G}_1, E_1) and (\tilde{G}_2, E_2) are two fuzzy soft UP_i -ideals of A . Since endurance $\in E_1 \cap E_2$, we have

$$\begin{aligned}
& (f_{\tilde{G}_1[\text{endurance}] \cup \tilde{G}_2[\text{endurance}]})(\text{black} \cdot \text{green}) = (f_{\tilde{G}_1[\text{endurance}] \cup \tilde{G}_2[\text{endurance}]})(\text{green}) = \\
& \quad 0.5 \not\geq 0.6 = \min\{0.6, 0.7\} = \\
& \min\{(f_{\tilde{G}_1[\text{endurance}] \cup \tilde{G}_2[\text{endurance}]})(\text{cyan}), (f_{\tilde{G}_1[\text{endurance}] \cup \tilde{G}_2[\text{endurance}]})(\text{blue})\} = \\
& \quad \min\{(f_{\tilde{G}_1[\text{endurance}] \cup \tilde{G}_2[\text{endurance}]})(\text{black} \cdot (\text{blue} \cdot \\
& \quad \text{green})), (f_{\tilde{G}_1[\text{endurance}] \cup \tilde{G}_2[\text{endurance}]})(\text{blue})\}.
\end{aligned}$$

Thus $\tilde{G}_1[\text{endurance}] \cup \tilde{G}_2[\text{endurance}]$ is not a fuzzy UP_i -ideal of A , that is, $(\tilde{G}_1, E_1) \cup (\tilde{G}_2, E_2)$ is not an endurance-fuzzy soft UP_i -ideal of A . Hence, $(\tilde{G}_1, E_1) \cup (\tilde{G}_2, E_2)$ is not a fuzzy soft UP_i -ideal of A . Moreover, $(\tilde{G}_1, E_1) \uplus (\tilde{G}_2, E_2)$ is not a fuzzy soft UP_i -ideal of A .

Definition 4.4.80 A fuzzy soft set (\tilde{F}, E) over A is called a *fuzzy soft strongly UP_s -ideal* based on $e \in E$ (we shortly call an *e-fuzzy soft strongly UP_s -ideal*) of A if a fuzzy set $\tilde{F}[e]$ in A is a fuzzy strongly UP_s -ideal of A . If (\tilde{F}, E) is an *e-fuzzy soft strongly UP_s -ideal* of A for all $e \in E$, we say that (\tilde{F}, E) is a *fuzzy soft strongly UP_s -ideal* of A .

Definition 4.4.81 A fuzzy soft set (\tilde{F}, E) over A is called a *constant fuzzy soft set* based on $e \in E$ (we shortly call an *e-constant fuzzy soft set*) of A if a fuzzy set $\tilde{F}[e]$ in A is constant. If (\tilde{F}, E) is an *e-constant fuzzy soft set* over A for all $e \in E$, we say that (\tilde{F}, E) is a *constant fuzzy soft set* over A .

Theorem 4.4.82 *Every e-fuzzy soft strongly UP_s -ideal of A is an e-fuzzy soft UP_s -ideal. Moreover, every fuzzy soft strongly UP_s -ideal of A is a fuzzy soft UP_s -ideal.*

Theorem 4.4.83 *e-fuzzy soft strongly UP_s -ideals and e-constant fuzzy soft sets coincide in A . Moreover, fuzzy soft strongly UP_s -ideals and constant fuzzy soft sets coincide in A .*

In the next theorem, we give necessary condition for fuzzy soft strongly UP_s -ideals of f -UP-semigroups.

Theorem 4.4.84 *If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.12) (or (4.3.13) or (4.3.14)) and (3.0.14), then (\tilde{F}, E) is a fuzzy soft strongly UP_s -ideal of A .*

Proof. It is straightforward by Propositions 4.3.25 (or 4.3.27 or 4.3.29) and Lemma 3.0.36 (1). \square

The following example shows that the converse of Theorem 4.4.82 is not true.

Example 4.4.85 Let A be a set of four brands of a pick-up truck, that is,

$$A = \{\text{Toyota Hilux}(\text{TH}), \text{Mitsubishi Triton}(\text{MT}), \text{Ford Ranger}(\text{FR}), \\ \text{Isuzu D-Max}(\text{ID})\}.$$

Define two binary operations \cdot and $*$ on A as the following Cayley tables:

\cdot	MT	FR	ID	TH	$*$	MT	FR	ID	TH
MT	MT	FR	ID	TH	MT	MT	MT	MT	MT
FR	MT	MT	ID	TH	FR	MT	FR	MT	MT
ID	MT	FR	MT	TH	ID	MT	MT	ID	MT
TH	MT	FR	ID	MT	TH	MT	TH	MT	MT

Then $A = (A, \cdot, *, \text{Mitsubishi Triton})$ is an f -UP-semigroup. Let (\tilde{F}, E) be a fuzzy soft set over A where

$$E := \{\text{displacement, horse power, torque}\}$$

with $\tilde{F}[\text{displacement}]$, $\tilde{F}[\text{horse power}]$, and $\tilde{F}[\text{torque}]$ are fuzzy sets in A defined

as follows:

\tilde{F}	MT	FR	ID	TH
displacement	1	0.6	0.4	0.7
horse power	0.9	0.6	0.5	0.5
torque	0.9	0.7	0.6	0.5

Then (\tilde{F}, E) is a torque-fuzzy soft UP_s -ideal of A but $\tilde{F}[\text{torque}]$ is not a fuzzy strongly UP_s -ideal of A . Indeed,

$$f_{\tilde{F}[\text{torque}]}(\text{ID}) = 0.6 \not\geq 0.7 = \min\{0.9, 0.7\} = \min\{f_{\tilde{F}[\text{torque}]}(\text{MT}), f_{\tilde{F}[\text{torque}]}(\text{FR})\} = \min\{f_{\tilde{F}[\text{torque}]}((\text{ID} \cdot \text{FR}) \cdot (\text{ID} \cdot \text{ID})), f_{\tilde{F}[\text{torque}]}(\text{FR})\}.$$

Hence, (\tilde{F}, E) is not a torque-fuzzy soft strongly UP_s -ideal of A .

The proof of the following theorem can be verified easily.

Theorem 4.4.86 *If (\tilde{F}, E) is a fuzzy soft strongly UP_s -ideal of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft strongly UP_s -ideal of A .*

By using Theorem 4.2.32, we can obtain the following two theorems in the same way as Theorems 4.4.8 and 4.4.9.

Theorem 4.4.87 *The extended intersection of two fuzzy soft strongly UP_s -ideals of A is also a fuzzy soft strongly UP_s -ideal. Moreover, the intersection of two fuzzy soft strongly UP_s -ideals of A is also a fuzzy soft strongly UP_s -ideal.*

Theorem 4.4.88 *The union of two fuzzy soft strongly UP_s -ideals is also a fuzzy soft strongly UP_s -ideal. Moreover, the restricted union of two fuzzy soft strongly UP_s -ideals of A is also a fuzzy soft strongly UP_s -ideal.*

Definition 4.4.89 A fuzzy soft set (\tilde{F}, E) over A is called a *fuzzy soft strongly UP_1 -ideal* based on $e \in E$ (we shortly call an *e-fuzzy soft strongly UP_1 -ideal*)

of A if a fuzzy set $\tilde{F}[e]$ in A is a fuzzy strongly UP_i -ideal of A . If (\tilde{F}, E) is an e -fuzzy soft strongly UP_i -ideal of A for all $e \in E$, we say that (\tilde{F}, E) is a *fuzzy soft strongly UP_i -ideal* of A .

Theorem 4.4.90 *Every e -fuzzy soft strongly UP_i -ideal of A is an e -fuzzy soft UP_i -ideal. Moreover, every fuzzy soft strongly UP_i -ideal of A is a fuzzy soft UP_i -ideal.*

Theorem 4.4.91 *e -fuzzy soft strongly UP_i -ideals and e -constant fuzzy soft sets coincide in A . Moreover, fuzzy soft strongly UP_i -ideals and constant fuzzy soft sets coincide in A .*

Corollary 4.4.92 *e -fuzzy soft strongly UP_s -ideals, e -fuzzy soft strongly UP_i -ideals, and e -constant fuzzy soft sets coincide in A . Moreover, fuzzy soft strongly UP_s -ideals, fuzzy soft strongly UP_i -ideals and constant fuzzy soft sets coincide in A .*

Proof. It is straightforward by Theorems 4.4.83 and 4.4.91. \square

In the next theorem, we give necessary condition for fuzzy soft strongly UP_i -ideals of f -UP-semigroups.

Theorem 4.4.93 *If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.12) (or (4.3.13) or (4.3.14)) and (3.0.15), then (\tilde{F}, E) is a fuzzy soft strongly UP_i -ideal of A .*

Proof. It is straightforward by Proposition 4.3.25 (or 4.3.27 or 4.3.29) and Lemma 3.0.36 (2). \square

The following example shows that the converse of Theorem 4.4.90 is not true.

Example 4.4.94 In Example 4.4.85, we know that (\tilde{F}, E) is a displacement-fuzzy soft UP_i -ideal of A but $\tilde{F}[\text{displacement}]$ is not a fuzzy strongly UP_i -ideal of A . Indeed,

$$\begin{aligned} f_{\tilde{F}[\text{displacement}]}(\text{ID}) &= 0.4 \not\geq 0.6 = \min\{1, 0.6\} = \\ &\min\{f_{\tilde{F}[\text{displacement}]}(\text{MT}), f_{\tilde{F}[\text{displacement}]}(\text{FR})\} = \\ &\min\{f_{\tilde{F}[\text{displacement}]}((\text{ID} \cdot \text{FR}) \cdot (\text{ID} \cdot \text{ID})), f_{\tilde{F}[\text{displacement}]}(\text{FR})\}. \end{aligned}$$

Hence, (\tilde{F}, E) is not a displacement-fuzzy soft strongly UP_i -ideal of A .

The proof of the following theorem can be verified easily.

Theorem 4.4.95 *If (\tilde{F}, E) is a fuzzy soft strongly UP_i -ideal of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft strongly UP_i -ideal of A .*

By using Theorem 4.2.33, we can obtain the following two theorems in the same way as Theorems 4.4.8 and 4.4.9.

Theorem 4.4.96 *The extended intersection of two fuzzy soft strongly UP_i -ideals of A is also a fuzzy soft strongly UP_i -ideal. Moreover, the intersection of two fuzzy soft strongly UP_i -ideals of A is also a fuzzy soft strongly UP_i -ideal.*

Theorem 4.4.97 *The union of two fuzzy soft strongly UP_i -ideals of A is also a fuzzy soft strongly UP_s -ideal. Moreover, the restricted union of two fuzzy soft strongly UP_i -ideals of A is also a fuzzy soft strongly UP_i -ideal.*

Then, we get the diagram of generalization of fuzzy soft sets over fully UP -semigroups as shown in Figure 4.4 below.

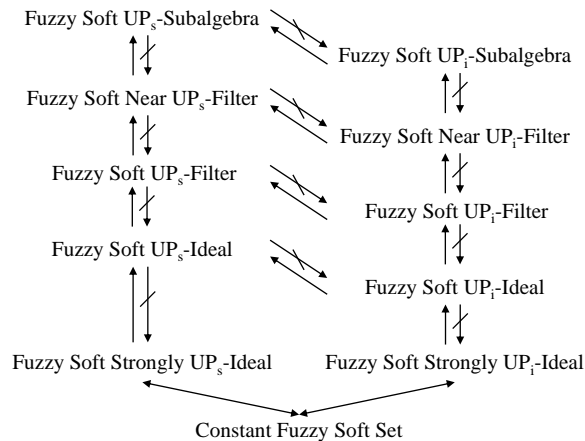


Figure 3: Fuzzy soft sets over fully UP-semigroups

4.5 Properties of operations for fuzzy soft sets over fully UP-semigroups

From now on, we shall let A be an f -UP-semigroup $A = (A, \cdot, *, 0)$ and P be a set of parameters. Let $\mathcal{F}(A)$ denotes the set of all fuzzy sets in A . A subset E of P is called a *set of statistics*.

Definition 4.5.1 [24] Let (\tilde{F}, E_1) and (\tilde{G}, E_2) be two fuzzy soft sets over a common universe U . The *OR* of (\tilde{F}, E_1) and (\tilde{G}, E_2) is defined to be the fuzzy soft set $(\tilde{F}, E_1) \vee (\tilde{G}, E_2) = (\tilde{H}, E)$ satisfying the following conditions:

- (i) $E = E_1 \times E_2$ and
- (ii) $\tilde{H}[e_1, e_2] = \tilde{F}[e_1] \cup \tilde{G}[e_2]$ for all $(e_1, e_2) \in E$.

Definition 4.5.2 [24] Let (\tilde{F}, E_1) and (\tilde{G}, E_2) be two fuzzy soft sets over a common universe U . The *AND* of (\tilde{F}, E_1) and (\tilde{G}, E_2) is defined to be the fuzzy soft set $(\tilde{F}, E_1) \wedge (\tilde{G}, E_2) = (\tilde{H}, E)$ satisfying the following conditions:

- (i) $E = E_1 \times E_2$ and
- (ii) $\tilde{H}[e_1, e_2] = \tilde{F}[e_1] \cap \tilde{G}[e_2]$ for all $(e_1, e_2) \in E$.

We will introduce the notions of the restricted union, the union, the intersection, the extended intersection, the AND, and the OR of any fuzzy soft sets and apply to f -UP-semigroups.

Definition 4.5.3 Let $\{(\tilde{F}_i, E_i) \mid i \in I\}$ be a nonempty family of fuzzy soft sets over a common universe U where I is an arbitrary index set. The *restricted union* of (\tilde{F}_i, E_i) is defined to be the fuzzy soft set $\bigcup_{i \in I} (\tilde{F}_i, E_i) = (\tilde{F}, E)$ satisfying the following conditions:

- (i) $E = \bigcap_{i \in I} E_i \neq \emptyset$ and
- (ii) $\tilde{F}[e] = \bigcup_{i \in I} \tilde{F}_i[e]$ for all $e \in E$.

Theorem 4.5.4 *The restricted union of family of fuzzy soft near UP_i -filters of A is also a fuzzy soft near UP_i -filter.*

Proof. Let (\tilde{F}_i, E_i) be a fuzzy soft near UP_i -filters of A for all $i \in I$. Assume that $\bigcup_{i \in I} (\tilde{F}_i, E_i) = (\tilde{F}, E)$ be the restricted union of (\tilde{F}_i, E_i) for all $i \in I$. Then $E = \bigcap_{i \in I} E_i \neq \emptyset$. Let $e \in E$. By Theorem 4.2.10, we have $\tilde{F}[e] = \bigcup_{i \in I} \tilde{F}_i[e]$ is a fuzzy near UP_i -filter of A . Therefore, (\tilde{F}, E) is an e -fuzzy soft near UP_i -filter of A . But since e is an arbitrary statistic of E , we have (\tilde{F}, E) is a fuzzy soft near UP_i -filter of A . \square

In the same way as Theorem 4.5.4, we can use Theorems 4.2.32 (resp., 4.2.33) to prove that the restricted union of family of fuzzy soft strongly UP_s -ideals (resp., fuzzy soft strongly UP_i -ideals) of A is also a fuzzy soft strongly UP_s -ideal (resp., fuzzy soft strongly UP_i -ideal).

Definition 4.5.5 Let $\{(\tilde{F}_i, E_i) \mid i \in I\}$ be a nonempty family of fuzzy soft sets over a common universe U where I is an arbitrary index set. The *union* of (\tilde{F}_i, E_i) is defined to be the fuzzy soft set $\bigcup_{i \in I} (\tilde{F}_i, E_i) = (\tilde{F}, E)$ satisfying the following conditions:

- (i) $E = \bigcup_{i \in I} E_i$ and
- (ii) $\tilde{F}[e] = \bigcup_{j \in J} \tilde{F}_j[e]$ for all $e \in E$ with $e \in \bigcap_{j \in J} E_j - \bigcup_{k \in I-J} E_k$ where $\emptyset \neq J \subseteq I$.

Theorem 4.5.6 *The union of family of fuzzy soft near UP_i -filters of A is also a fuzzy soft near UP_i -filter.*

Proof. Let (\tilde{F}_i, E_i) be a fuzzy soft near UP_i -filters of A for all $i \in I$. Assume that $\bigcap_{i \in I} (\tilde{F}_i, E_i) = (\tilde{F}, E)$ be the union of (\tilde{F}_i, E_i) for all $i \in I$. Then $E = \bigcup_{i \in I} E_i$. Let $e \in E$.

Case 1: $|J| = |I|$. By Theorem 4.5.4, we have $\tilde{F}[e] = \bigcap_{i \in I} \tilde{F}_i[e]$ is a fuzzy near UP_i -filter of A .

Case 2: $|J| = 1$, that is, J is a singleton set. Then $\tilde{F}[e] = \bigcap_{j \in \{j\}} \tilde{F}_j[e] = \tilde{F}_j[e]$ is a fuzzy near UP_i -filter of A .

Case 3: $1 < |J| < |I|$. Then $\tilde{F}[e] = \bigcap_{j \in J} \tilde{F}_j[e]$. Since $e \in E_j$ for all $j \in J$ and $e \notin E_k$ for some $k \in I - J$ and by same Case 1, we have $\tilde{F}[e]$ is a fuzzy near UP_i -filter of A .

Therefore, (\tilde{F}, E) is an e -fuzzy soft near UP_i -filter of A . But since e is an arbitrary statistic of E , we have (\tilde{F}, E) is a fuzzy soft near UP_i -filter of A . \square

In the same way as Theorem 4.5.6, we can prove that the union of family of fuzzy soft strongly UP_s -ideals (resp., fuzzy soft strongly UP_i -ideals) of A is also a fuzzy soft strongly UP_s -ideal (resp., fuzzy soft strongly UP_i -ideal).

In section 4.4, we show that the union of two fuzzy soft UP_s -subalgebras (resp., fuzzy soft UP_i -subalgebras, fuzzy soft near UP_s -filters, fuzzy soft UP_s -filters, fuzzy soft UP_i -filters, fuzzy soft UP_s -ideals, fuzzy soft UP_i -ideals) of A is not fuzzy soft UP_s -subalgebra (resp., fuzzy soft UP_i -subalgebra, fuzzy soft near

UP_s-filter, fuzzy soft UP_s-filter, fuzzy soft UP_i-filter, fuzzy soft UP_s-ideal, fuzzy soft UP_i-ideal).

Definition 4.5.7 Let $\{(\tilde{F}_i, E_i) \mid i \in I\}$ be a nonempty family of fuzzy soft sets over a common universe U where I is an arbitrary index set. The *intersection* of (\tilde{F}_i, E_i) is defined to be the fuzzy soft set $\bigcap_{i \in I} (\tilde{F}_i, E_i) = (\tilde{F}, E)$ satisfying the following conditions:

- (i) $E = \bigcap_{i \in I} E_i \neq \emptyset$ and
- (ii) $\tilde{F}[e] = \bigcap_{i \in I} \tilde{F}_i[e]$ for all $e \in E$.

Theorem 4.5.8 *The intersection of family of fuzzy soft UP_s-subalgebras of A is also a fuzzy soft UP_s-subalgebra.*

Proof. Let (\tilde{F}_i, E_i) be a fuzzy soft UP_s-subalgebras of A for all $i \in I$. Assume that $\bigcap_{i \in I} (\tilde{F}_i, E_i) = (\tilde{F}, E)$ is the intersection of (\tilde{F}_i, E_i) for all $i \in I$. Then $E = \bigcap_{i \in I} E_i \neq \emptyset$. Let $e \in E$. By Theorem 4.2.2, we have $\tilde{F}[e] = \bigcap_{i \in I} \tilde{F}_i[e]$ is a fuzzy UP_s-subalgebra of A . Therefore, (\tilde{F}, E) is an e -fuzzy soft UP_s-subalgebra of A . But since e is an arbitrary statistic of E , we have (\tilde{F}, E) is a fuzzy soft UP_s-subalgebra of A . \square

In the same way as Theorem 4.5.8, we can use Theorems 4.2.4 (resp., 4.2.7, 4.2.9, 4.2.15, 4.2.17, 4.2.24, 4.2.26, 4.2.32, 4.2.33) to prove that the intersection of family of fuzzy soft UP_i-subalgebras (resp., fuzzy soft near UP_s-filters, fuzzy soft near UP_i-filters, fuzzy soft UP_s-filters, fuzzy soft UP_i-filters, fuzzy soft UP_s-ideals, fuzzy soft UP_i-ideals, fuzzy soft strongly UP_s-ideals, fuzzy soft strongly UP_i-ideals) of A is also a fuzzy soft UP_i-subalgebra (resp., fuzzy soft near UP_s-filter, fuzzy soft near UP_i-filter, fuzzy soft UP_s-filter, fuzzy soft UP_i-filter, fuzzy soft UP_s-ideal, fuzzy soft UP_i-ideal, fuzzy soft strongly UP_s-ideal, fuzzy soft strongly UP_i-ideal).

Definition 4.5.9 Let $\{(\tilde{F}_i, E_i) \mid i \in I\}$ be a nonempty family of fuzzy soft sets over a common universe U where I is an arbitrary index set. The *extended intersection* of (\tilde{F}_i, E_i) is defined to be the fuzzy soft set $\bigcap_{i \in I} (\tilde{F}_i, E_i) = (\tilde{F}, E)$ satisfying the following conditions:

- (i) $E = \bigcup_{i \in I} E_i$ and
- (ii) $\tilde{F}[e] = \bigcap_{j \in J} \tilde{F}_j[e]$ for all $e \in E$ with $e \in \bigcap_{j \in J} E_j - \bigcup_{k \in I-J} E_k$ where $\emptyset \neq J \subseteq I$.

Theorem 4.5.10 *The extended intersection of family of fuzzy soft UP_s -subalgebras of A is also a fuzzy soft UP_s -subalgebra.*

Proof. Let (\tilde{F}_i, E_i) be a fuzzy soft UP_s -subalgebras of A for all $i \in I$. Assume that $\bigcap_{i \in I} (\tilde{F}_i, E_i) = (\tilde{F}, E)$ is the extended intersection of (\tilde{F}_i, E_i) for all $i \in I$. Then $E = \bigcup_{i \in I} E_i$. Let $e \in E$.

Case 1: $|J| = |I|$. By Theorem 4.5.8, we have $\tilde{F}[e] = \bigcap_{i \in I} \tilde{F}_i[e]$ is a fuzzy UP_s -subalgebra of A .

Case 2: $|J| = 1$, that is, J is a singleton set. Then $\tilde{F}[e] = \bigcap_{j \in \{j\}} \tilde{F}_j[e] = \tilde{F}_j[e]$ is a fuzzy UP_s -subalgebra of A .

Case 3: $1 < |J| < |I|$. Then $\tilde{F}[e] = \bigcap_{j \in J} \tilde{F}_j[e]$. Since $e \in E_j$ for all $j \in J$ and $e \notin E_k$ for some $k \in I - J$ and by same Case 1, we have $\tilde{F}[e]$ is a fuzzy UP_s -subalgebra of A .

Therefore, (\tilde{F}, E) is an e -fuzzy soft UP_s -subalgebra of A . But since e is an arbitrary statistic of E , we have (\tilde{F}, E) is a fuzzy soft UP_s -subalgebra of A . \square

In the same way as Theorem 4.5.10, we can prove that the extended intersection of family of fuzzy soft UP_i -subalgebras (resp., fuzzy soft near UP_s -

filters, fuzzy soft near UP_i -filters, fuzzy soft UP_s -filters, fuzzy soft UP_i -filters, fuzzy soft UP_s -ideals, fuzzy soft UP_i -ideals, fuzzy soft strongly UP_s -ideals, fuzzy soft strongly UP_i -ideals) of A is also a fuzzy soft UP_i -subalgebra (resp., fuzzy soft near UP_s -filter, fuzzy soft near UP_i -filter, fuzzy soft UP_s -filter, fuzzy soft UP_i -filter, fuzzy soft UP_s -ideal, fuzzy soft UP_i -ideal, fuzzy soft strongly UP_s -ideal, fuzzy soft strongly UP_i -ideal).

Definition 4.5.11 Let $\{(\tilde{F}_i, E_i) \mid i \in I\}$ be a nonempty family of fuzzy soft sets over a common universe U where I is an arbitrary index set. The AND of (\tilde{F}_i, E_i) is defined to be the fuzzy soft set $\bigwedge_{i \in I} (\tilde{F}_i, E_i) = (\tilde{F}, E)$ satisfying the following conditions:

- (i) $E = \prod_{i \in I} E_i$ and
- (ii) $\tilde{F}[(e_i)_{i \in I}] = \bigcap_{i \in I} \tilde{F}_i[e_i]$ for all $(e_i)_{i \in I} \in E$.

Theorem 4.5.12 The AND of family of fuzzy soft UP_s -subalgebras of A is also a fuzzy soft UP_s -subalgebra.

Proof. Let (\tilde{F}_i, E_i) be a fuzzy soft UP_s -subalgebras of A for all $i \in I$. By means of Definition 4.5.11, we assume that $\bigwedge_{i \in I} (\tilde{F}_i, E_i) = (\tilde{F}, E)$ such that $E = \prod_{i \in I} E_i$ and $\tilde{F}[(e_i)_{i \in I}] = \bigcap_{i \in I} \tilde{F}_i[e_i]$ for all $(e_i)_{i \in I} \in E$. Assume that $e = (e_i)_{i \in I} \in E$ and let $x, y \in A$. Then

$$\begin{aligned}
 f_{\tilde{F}[e]}(x \cdot y) &= f_{\bigcap_{i \in I} \tilde{F}_i[e_i]}(x \cdot y) \\
 &= \inf \{f_{\tilde{F}_i[e_i]}(x \cdot y)\}_{i \in I} \\
 &\geq \inf \{\min \{f_{\tilde{F}_i[e_i]}(x), f_{\tilde{F}_i[e_i]}(y)\}\}_{i \in I} \\
 &= \min \{\inf \{f_{\tilde{F}_i[e_i]}(x)\}_{i \in I}, \inf \{f_{\tilde{F}_i[e_i]}(y)\}_{i \in I}\} \\
 &= \min \{f_{\bigcap_{i \in I} \tilde{F}_i[e_i]}(x), f_{\bigcap_{i \in I} \tilde{F}_i[e_i]}(y)\} \\
 &= \min \{f_{\tilde{F}[e]}(x), f_{\tilde{F}[e]}(y)\}, \text{ and}
 \end{aligned}$$

$$\begin{aligned}
f_{\tilde{F}[e]}(x * y) &= f_{\bigcap_{i \in I} \tilde{F}_i[e_i]}(x * y) \\
&= \inf \{f_{\tilde{F}_i[e_i]}(x * y)\}_{i \in I} \\
&\geq \inf \{\min \{f_{\tilde{F}_i[e_i]}(x), f_{\tilde{F}_i[e_i]}(y)\}\}_{i \in I} \\
&= \min \{\inf \{f_{\tilde{F}_i[e_i]}(x)\}_{i \in I}, \inf \{f_{\tilde{F}_i[e_i]}(y)\}_{i \in I}\} \\
&= \min \{f_{\bigcap_{i \in I} \tilde{F}_i[e_i]}(x), f_{\bigcap_{i \in I} \tilde{F}_i[e_i]}(y)\} \\
&= \min \{f_{\tilde{F}[e]}(x), f_{\tilde{F}[e]}(y)\}.
\end{aligned}$$

Therefore, $\tilde{F}[e]$ is a fuzzy UP_s -subalgebra of A , that is, (\tilde{F}, E) is an e -fuzzy soft UP_s -subalgebra of A . But since e is an arbitrary statistic of E , we have (\tilde{F}, E) is a fuzzy soft UP_s -subalgebra of A . \square

In the same way as Theorem 4.5.12, we can prove that the AND of family of fuzzy soft UP_i -subalgebras (resp., fuzzy soft near UP_s -filters, fuzzy soft near UP_i -filters, fuzzy soft UP_s -filters, fuzzy soft UP_i -filters, fuzzy soft UP_s -ideals, fuzzy soft UP_i -ideals, fuzzy soft strongly UP_s -ideals, fuzzy soft strongly UP_i -ideals) of A is also a fuzzy soft UP_i -subalgebra (resp., fuzzy soft near UP_s -filter, fuzzy soft near UP_i -filter, fuzzy soft UP_s -filter, fuzzy soft UP_i -filter, fuzzy soft UP_s -ideal, fuzzy soft UP_i -ideal, fuzzy soft strongly UP_s -ideal, fuzzy soft strongly UP_i -ideal).

Definition 4.5.13 Let $\{(\tilde{F}_i, E_i) \mid i \in I\}$ be a nonempty family of fuzzy soft sets over a common universe U where I is an arbitrary index set. The OR of (\tilde{F}_i, E_i) is defined to be the fuzzy soft set $\bigvee_{i \in I} (\tilde{F}_i, E_i) = (\tilde{F}, E)$ satisfying the following conditions:

- (i) $E = \prod_{i \in I} E_i$ and
- (ii) $\tilde{F}[(e_i)_{i \in I}] = \bigcup_{i \in I} \tilde{F}_i[e_i]$ for all $(e_i)_{i \in I} \in E$.

Theorem 4.5.14 *The OR of family of fuzzy soft near UP_i -filters of A is also a fuzzy soft near UP_i -filter.*

Proof. Let (\tilde{F}_i, E_i) be a fuzzy soft near UP_i -filters of A for all $i \in I$. By means of Definition 4.5.13, we assume that $\bigvee_{i \in I} (\tilde{F}_i, E_i) = (\tilde{F}, E)$ such that $E = \prod_{i \in I} E_i$ and $\tilde{F}[(e_i)_{i \in I}] = \bigcup_{i \in I} \tilde{F}_i[e_i]$ for all $(e_i)_{i \in I} \in E$. Assume that $e = (e_i)_{i \in I} \in E$ and let $x, y \in A$. Then

$$\begin{aligned}
f_{\tilde{F}[e]}(0) &= f_{\bigcup_{i \in I} \tilde{F}_i[e_i]}(0) \\
&= \sup\{f_{\tilde{F}_i[e_i]}(0)\}_{i \in I} \\
&\geq \sup\{f_{\tilde{F}_i[e_i]}(x)\}_{i \in I} \\
&= f_{\bigcup_{i \in I} \tilde{F}_i[e_i]}(x) \\
&= f_{\tilde{F}[e]}(x), \\
f_{\tilde{F}[e]}(x \cdot y) &= f_{\bigcup_{i \in I} \tilde{F}_i[e_i]}(x \cdot y) \\
&= \sup\{f_{\tilde{F}_i[e_i]}(x \cdot y)\}_{i \in I} \\
&\geq \sup\{f_{\tilde{F}_i[e_i]}(y)\}_{i \in I} \\
&= f_{\bigcup_{i \in I} \tilde{F}_i[e_i]}(y) \\
&= f_{\tilde{F}[e]}(y), \text{ and} \\
f_{\tilde{F}[e]}(x * y) &= f_{\bigcup_{i \in I} \tilde{F}_i[e_i]}(x * y) \\
&= \sup\{f_{\tilde{F}_i[e_i]}(x * y)\}_{i \in I} \\
&\geq \sup\{\max\{f_{\tilde{F}_i[e_i]}(x), f_{\tilde{F}_i[e_i]}(y)\}\}_{i \in I} \\
&= \max\{\sup\{f_{\tilde{F}_i[e_i]}(x)\}_{i \in I}, \sup\{f_{\tilde{F}_i[e_i]}(y)\}_{i \in I}\} \\
&= \max\{f_{\bigcap_{i \in I} \tilde{F}_i[e_i]}(x), f_{\bigcap_{i \in I} \tilde{F}_i[e_i]}(y)\} \\
&= \max\{f_{\tilde{F}[e]}(x), f_{\tilde{F}[e]}(y)\}.
\end{aligned}$$

Therefore, $\tilde{F}[e]$ is a fuzzy near UP_i -filter of A , that is, (\tilde{F}, E) is an e -fuzzy soft near UP_i -filter of A . But since e is an arbitrary statistic of E , we have (\tilde{F}, E) is

a fuzzy soft near UP_i -filter of A . \square

In the same way as Theorem 4.5.14, we can prove that the OR of family of fuzzy soft strongly UP_s -ideals (resp., fuzzy soft strongly UP_i -ideals) of A is also a fuzzy soft strongly UP_s -ideal (resp., fuzzy soft strongly UP_i -ideal).

The following example shows that the OR of two fuzzy soft UP_s -subalgebras of A are not fuzzy soft UP_s -subalgebra.

Example 4.5.15 By Cayley tables in Example 4.4.7, we know that $A = (A, \cdot, *, X)$ is an f -UP-semigroup. Let (\tilde{F}_1, E_1) and (\tilde{F}_2, E_2) be two fuzzy soft sets over A where

$$E_1 := \{\text{price, beauty, specifications}\} \text{ and } E_2 := \{\text{price, stability}\}$$

with $\tilde{F}_1[\text{price}]$, $\tilde{F}_1[\text{beauty}]$, $\tilde{F}_1[\text{specifications}]$, $\tilde{F}_2[\text{price}]$, and $\tilde{F}_2[\text{stability}]$ are fuzzy sets in A defined as follows:

\tilde{F}_1	X	7	6	5	\tilde{F}_2	X	7	6	5
price	0.9	0.7	0.9	0.2	price	0.9	0.3	0.2	0.8
beauty	1	0.8	0.3	0.2	stability	0.7	0.2	0.5	0.2
specifications	0.6	0.5	0.3	0.4					

Then (\tilde{F}_1, E_1) and (\tilde{F}_2, E_2) are two fuzzy soft UP_s -subalgebras of A . Since $(\text{price, price}) \in E_1 \times E_2$, we have

$$\begin{aligned}
 (f_{\tilde{F}_1[\text{price}] \cup \tilde{F}_2[\text{price}]})(5 * 6) &= (f_{\tilde{F}_1[\text{price}] \cup \tilde{F}_2[\text{price}]})(7) \\
 &= 0.7 \\
 &\neq 0.8 \\
 &= \min\{0.8, 0.9\}
 \end{aligned}$$

$$= \min\{(f_{\tilde{F}_1[\text{price}] \cup \tilde{F}_2[\text{price}]}) (5), (f_{\tilde{F}_1[\text{price}] \cup \tilde{F}_2[\text{price}]}) (6)\}.$$

Thus $\tilde{F}_1[\text{price}] \cup \tilde{F}_2[\text{price}]$ is not a fuzzy UP_s -subalgebra of A , that is, $(\tilde{F}_1, E_1) \cup (\tilde{F}_2, E_2)$ is not a (price, price)-fuzzy soft UP_s -subalgebra of A . Hence, $(\tilde{F}_1, E_1) \cup (\tilde{F}_2, E_2)$ is not a fuzzy soft UP_s -subalgebra of A . Moreover, $(\tilde{F}_1, E_1) \vee (\tilde{F}_2, E_2)$ is not a fuzzy soft UP_s -subalgebra of A .

We can apply this example for check that the OR of two fuzzy soft UP_i -subalgebras (resp., fuzzy soft near UP_s -filters, fuzzy soft UP_s -filters, fuzzy soft UP_i -filters, fuzzy soft UP_s -ideals, fuzzy soft UP_i -ideals) of A are not fuzzy soft UP_i -subalgebra (resp., fuzzy soft near UP_s -filter, fuzzy soft UP_s -filter, fuzzy soft UP_i -filter, fuzzy soft UP_s -ideal, fuzzy soft UP_i -ideal).

We prove that certain distributive laws hold in fuzzy soft set theory with respect to the restricted union, the union, the intersection, and the extended intersection on any fuzzy soft sets.

Theorem 4.5.16 *Let (\tilde{F}_i, E_i) and (\tilde{F}, E) be fuzzy soft sets over a common universe U where I is a nonempty set. Then the following properties hold:*

- (1) $(\tilde{F}, E) \cap (\bigcup_{i \in I} (\tilde{F}_i, E_i)) = \bigcup_{i \in I} ((\tilde{F}, E) \cap (\tilde{F}_i, E_i)),$
- (2) $(\bigcup_{i \in I} (\tilde{F}_i, E_i)) \cap (\tilde{F}, E) = \bigcup_{i \in I} ((\tilde{F}_i, E_i) \cap (\tilde{F}, E)),$
- (3) $(\tilde{F}, E) \cup (\bigcap_{i \in I} (\tilde{F}_i, E_i)) = \bigcap_{i \in I} ((\tilde{F}, E) \cup (\tilde{F}_i, E_i)),$
- (4) $(\bigcap_{i \in I} (\tilde{F}_i, E_i)) \cup (\tilde{F}, E) = (\tilde{F}_i, E_i) \cup \bigcap_{i \in I} ((\tilde{F}, E)),$
- (5) $(\tilde{F}, E) \cup (\bigcup_{i \in I} (\tilde{F}_i, E_i)) = \bigcup_{i \in I} ((\tilde{F}, E) \cup (\tilde{F}_i, E_i)),$
- (6) $(\bigcup_{i \in I} (\tilde{F}_i, E_i)) \cup (\tilde{F}, E) = \bigcup_{i \in I} ((\tilde{F}_i, E_i) \cup (\tilde{F}, E)),$
- (7) $(\tilde{F}, E) \cup (\bigcap_{i \in I} (\tilde{F}_i, E_i)) = \bigcap_{i \in I} ((\tilde{F}, E) \cup (\tilde{F}_i, E_i)),$

$$(8) \quad (\bigcap_{i \in I} (\tilde{F}_i, E_i)) \cup (\tilde{F}, E) = \bigcap_{i \in I} ((\tilde{F}_i, E_i) \cup (\tilde{F}, E)),$$

$$(9) \quad (\tilde{F}, E) \cap (\bigcup_{i \in I} (\tilde{F}_i, E_i)) = \bigcup_{i \in I} ((\tilde{F}, E) \cap (\tilde{F}_i, E_i)),$$

$$(10) \quad (\bigcup_{i \in I} (\tilde{F}_i, E_i)) \cap (\tilde{F}, E) = \bigcup_{i \in I} ((\tilde{F}_i, E_i) \cap (\tilde{F}, E)),$$

$$(11) \quad (\tilde{F}, E) \cup (\bigcap_{i \in I} (\tilde{F}_i, E_i)) = \bigcap_{i \in I} ((\tilde{F}, E) \cup (\tilde{F}_i, E_i)), \text{ and}$$

$$(12) \quad (\bigcap_{i \in I} (\tilde{F}_i, E_i)) \cup (\tilde{F}, E) = \bigcap_{i \in I} ((\tilde{F}_i, E_i) \cup (\tilde{F}, E)).$$

Proof. (1) First, we investigate left hand side of the equality. Suppose that $\bigcup_{i \in I} (\tilde{F}_i, E_i) = (\tilde{G}, E^U)$ is the union of (\tilde{F}_i, E_i) for all $i \in I$. Then $E^U = \bigcup_{i \in I} E_i$ and for any $e \in E^U$, $\tilde{G}[e] = \bigcup_{j \in J} \tilde{F}_j[e]$ with $e \in \bigcap_{j \in J} E_j - \bigcup_{k \in I-J} E_k$ where $\emptyset \neq J \subseteq I$. Thus $(\tilde{F}, E) \cap (\bigcup_{i \in I} (\tilde{F}_i, E_i)) = (\tilde{F}, E) \cap (\tilde{G}, E^U) = (\tilde{H}, E^{UI})$. For any $e \in E^{UI} = E \cap E^U \neq \emptyset$, $\tilde{H}[e] = \tilde{F}[e] \cap \tilde{G}[e]$ where $E \cap E^U = E \cap (\bigcup_{i \in I} E_i) = \bigcup_{i \in I} (E \cap E_i)$. By considering \tilde{G} as piecewise defined function, we have $\tilde{H}[e] = \tilde{F}[e] \cap (\bigcup_{j \in J} \tilde{F}_j[e])$ with $e \in \bigcap_{j \in J} (E \cap E_j) - \bigcup_{k \in I-J} (E \cap E_k)$ where $\emptyset \neq J \subseteq I$.

Consider the right hand side of the equality. Suppose that $(\tilde{F}, E) \cap (\tilde{F}_i, E_i) = (\tilde{I}_i, E_i^I)$ is the intersection of (\tilde{F}, E) and (\tilde{F}_i, E_i) for all $i \in I$. Then $E_i^I = E \cap E_i \neq \emptyset$ and for any $e \in E_i^I$, $\tilde{I}_i[e] = \tilde{F}[e] \cap \tilde{F}_i[e]$. Now, $\bigcup_{i \in I} ((\tilde{F}, E) \cap (\tilde{F}_i, E_i)) = \bigcup_{i \in I} (\tilde{I}_i, E_i^I) = (\tilde{J}, E^{IU})$, where $E^{IU} = \bigcup_{i \in I} E_i^I = \bigcup_{i \in I} (E \cap E_i)$. For any $e \in E^{IU}$, $\tilde{J}[e] = \bigcup_{j \in J} \tilde{I}_j[e]$ with $e \in \bigcap_{j \in J} E_j^I - \bigcup_{k \in I-J} E_k^I$ where $\emptyset \neq J \subseteq I$. Considering \tilde{I}_i as piecewise functions for all $i \in I$, we have $\tilde{J}[e] = \bigcup_{j \in J} (\tilde{F}[e] \cap \tilde{F}_j[e])$ with $e \in \bigcap_{j \in J} (E \cap E_j) - \bigcup_{k \in I-J} (E \cap E_k)$ where $\emptyset \neq J \subseteq I$. By Theorem 3.0.37(1), it is clear that \tilde{H} and \tilde{J} are same set-valued mapping. Hence, $(\tilde{F}, E) \cap (\bigcup_{i \in I} (\tilde{F}_i, E_i)) = \bigcup_{i \in I} ((\tilde{F}, E) \cap (\tilde{F}_i, E_i))$.

(2) By using techniques as in (1) and by Theorem 3.0.37(2), then (2) can be derived.

(3) By using techniques as in (1) and by Theorem 3.0.37(3), then (3) can be derived.

(4) By using techniques as in (1) and by Theorem 3.0.37(4), then (4) can be derived.

(5) First, we investigate left hand side of the equality. Suppose that $\bigcup_{i \in I} (\tilde{F}_i, E_i) = (\tilde{G}, E^{RU})$ is the restricted union of (\tilde{F}_i, E_i) for all $i \in I$. Then $E^{RU} = \bigcap_{i \in I} E_i \neq \emptyset$ and for any $e \in E^{RU}$, $\tilde{G}[e] = \bigcup_{i \in I} \tilde{F}_i[e]$. Thus $(\tilde{F}, E) \cap (\bigcup_{i \in I} (\tilde{F}_i, E_i)) = (\tilde{F}, E) \cap (\tilde{G}, E^{RU}) = (\tilde{H}, E^{RUEI})$. For any $e \in E^{RUEI} = E \cup E^{RU}$, we have

$$\tilde{H}[e] = \begin{cases} \tilde{F}[e] & \text{if } e \in E \setminus E^{RU} \\ \tilde{G}[e] & \text{if } e \in E^{RU} \setminus E \\ \tilde{F}[e] \cap \tilde{G}[e] & \text{if } e \in E \cap E^{RU}. \end{cases}$$

By taking into account the definition of \tilde{G} along with \tilde{H} , we can write

$$\tilde{H}[e] = \begin{cases} \tilde{F}[e] & \text{if } e \in E \setminus (\bigcap_{i \in I} E_i) \\ \bigcup_{i \in I} \tilde{F}_i[e] & \text{if } e \in (\bigcap_{i \in I} E_i) \setminus E \\ \tilde{F}[e] \cap (\bigcup_{i \in I} \tilde{F}_i[e]) & \text{if } e \in E \cap (\bigcap_{i \in I} E_i). \end{cases}$$

Consider the right hand side of the equality. Suppose that $(\tilde{F}, E) \cap (\tilde{F}_i, E_i) = (\tilde{I}_i, E_i^{EI})$ is the extended intersection of (\tilde{F}, E) and (\tilde{F}_i, E_i) for all $i \in I$. Then for any $e \in E_i^{EI} = E \cup E_i$, we have

$$\tilde{I}_i[e] = \begin{cases} \tilde{F}[e] & \text{if } e \in E \setminus E_i \\ \tilde{F}_i[e] & \text{if } e \in E_i \setminus E \\ \tilde{F}[e] \cap \tilde{F}_i[e] & \text{if } e \in E \cap E_i. \end{cases}$$

Now, $\bigcup_{i \in I} ((\tilde{F}, E) \cap (\tilde{F}_i, E_i)) = \bigcup_{i \in I} (\tilde{I}_i, E_i^{EI}) = (\tilde{J}, E^{EIRU})$ where $E^{EIRU} = \bigcap_{i \in I} E_i^I = \bigcap_{i \in I} (E \cup E_i) = E \cup (\bigcap_{i \in I} E_i) \neq \emptyset$. For any $e \in E^{EIRU}$, $\tilde{J}[e] = \bigcup_{i \in I} \tilde{I}_i[e]$. By taking into account the properties of operations in set theory and considering \tilde{I}_i

as piecewise defined functions for all $i \in I$, we have

$$\tilde{J}[e] = \begin{cases} \bigcup_{i \in I} \tilde{F}[e] & \text{if } e \in E \setminus (\bigcap_{i \in I} E_i) \\ \bigcup_{i \in I} \tilde{F}_i[e] & \text{if } e \in (\bigcap_{i \in I} E_i) \setminus E \\ \bigcup_{i \in I} (\tilde{F}[e] \cap \tilde{F}_i[e]) & \text{if } e \in E \cap (\bigcap_{i \in I} E_i). \end{cases}$$

And so

$$\tilde{J}[e] = \begin{cases} \tilde{F}[e] & \text{if } e \in E \setminus (\bigcap_{i \in I} E_i) \\ \bigcup_{i \in I} \tilde{F}_i[e] & \text{if } e \in (\bigcap_{i \in I} E_i) \setminus E \\ \bigcup_{i \in I} (\tilde{F}[e] \cap \tilde{F}_i[e]) & \text{if } e \in E \cap (\bigcap_{i \in I} E_i). \end{cases}$$

By Theorem 3.0.37(1), it is clear that \tilde{H} and \tilde{J} are same set-valued mapping.

Hence, $(\tilde{F}, E) \cap (\bigcup_{i \in I} (\tilde{F}_i, E_i)) = \bigcup_{i \in I} ((\tilde{F}, E) \cap (\tilde{F}_i, E_i))$.

(6) By using techniques as in (5) and by Theorem 3.0.37(2), then (6) can is derived.

(7) By using techniques as in (5) and by Theorem 3.0.37(3), then (7) can is derived.

(8) By using techniques as in (5) and by Theorem 3.0.37(4), then (8) can is derived.

(9) First, we investigate left hand side of the equality. Suppose that $\bigcup_{i \in I} (\tilde{F}_i, E_i) = (\tilde{G}, E^{RU})$ is the restricted union of (\tilde{F}_i, E_i) for all $i \in I$. Then $E^{RU} = \bigcap_{i \in I} E_i \neq \emptyset$ and for any $e \in E^{RU}$, $\tilde{G}[e] = \bigcup_{i \in I} \tilde{F}_i[e]$. Thus $(\tilde{F}, E) \pitchfork (\bigcup_{i \in I} (\tilde{F}_i, E_i)) = (\tilde{F}, E) \pitchfork (\tilde{G}, E^{RU}) = (\tilde{H}, E^{RUI})$. For any $e \in E^{RUI} = E \cap E^{RU} = E \cap (\bigcap_{i \in I} E_i) \neq \emptyset$, we have $\tilde{H}[e] = \tilde{F}[e] \cap \tilde{G}[e] = \tilde{F}[e] \cap (\bigcup_{i \in I} \tilde{F}_i[e])$.

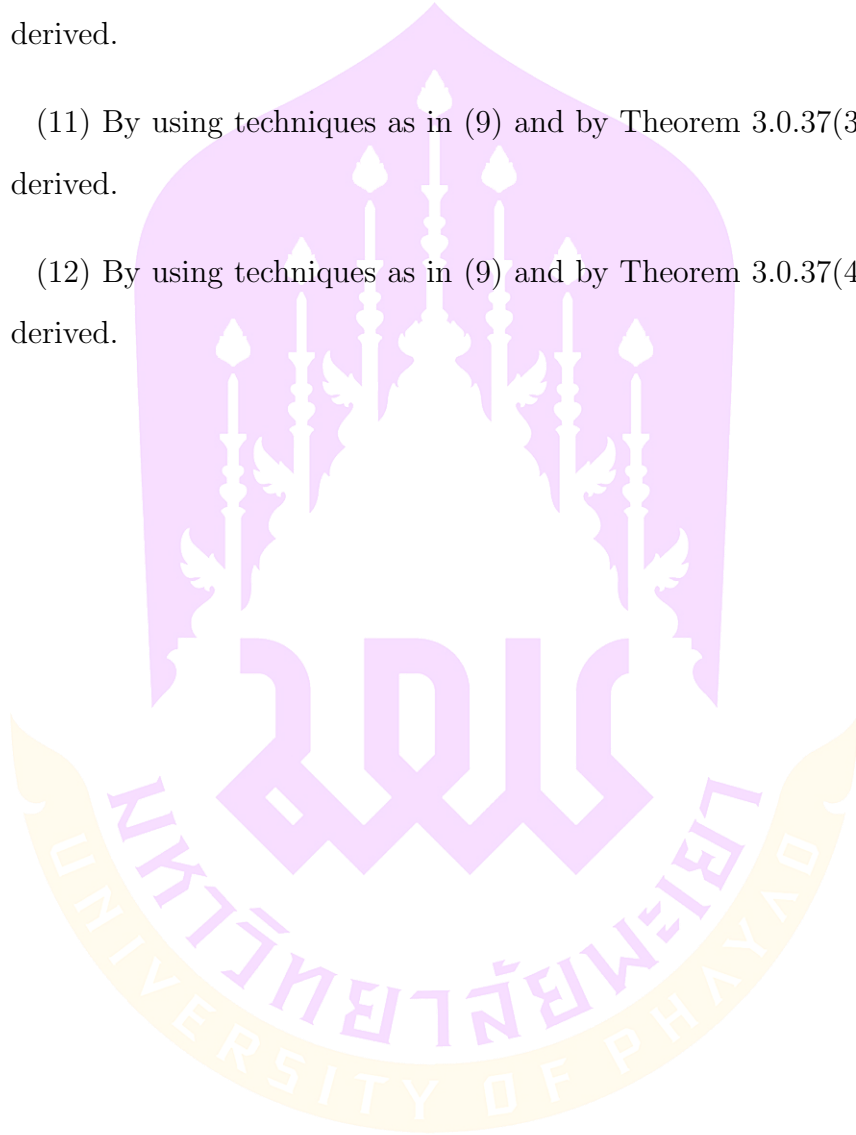
Consider the right hand side of the equality. Suppose that $(\tilde{F}, E) \pitchfork (\tilde{F}_i, E_i) = (\tilde{I}_i, E_i^I)$ is the intersection of (\tilde{F}, E) and (\tilde{F}_i, E_i) for all $i \in I$. Then $E_i^I = E \cap E_i \neq \emptyset$ and for any $e \in E_i^I$, $\tilde{I}_i[e] = \tilde{F}[e] \cap \tilde{F}_i[e]$. Now, $\bigcup_{i \in I} ((\tilde{F}, E) \pitchfork (\tilde{F}_i, E_i)) = \bigcup_{i \in I} (\tilde{I}_i, E_i^I) = (\tilde{J}, E^{IRU})$, where $E^{IRU} = \bigcap_{i \in I} E_i^I = \bigcap_{i \in I} (E \cap E_i) \neq \emptyset$. For any $e \in$

E^{IRU} , $\tilde{J}[e] = \bigcup_{j \in J} \tilde{I}_j[e] = \bigcup_{j \in J} (\tilde{F}[e] \cap \tilde{F}_j[e])$. Since $\bigcap_{i \in I} (E \cap E_i) = E \cap (\bigcap_{i \in I} E_i)$, we have $E^{IRU} = E^{RUI}$. By Theorem 3.0.37(1), it is clear that \tilde{H} and \tilde{J} are same set-valued mapping. Hence, $(\tilde{F}, E) \pitchfork (\bigcup_{i \in I} (\tilde{F}_i, E_i)) = \bigcup_{i \in I} ((\tilde{F}, E) \pitchfork (\tilde{F}_i, E_i))$.

(10) By using techniques as in (9) and by Theorem 3.0.37(2), then (10) can be derived.

(11) By using techniques as in (9) and by Theorem 3.0.37(3), then (11) can be derived.

(12) By using techniques as in (9) and by Theorem 3.0.37(4), then (12) can be derived. \square



CHAPTER V

CONCLUSIONS

From the study, we get the following results.

1. Every UP_i -subalgebra of A is a UP_s -subalgebra of A .
2. Every near UP_s -filter of A is a UP_s -subalgebra of A .
3. Every near UP_i -filter of A is a UP_i -subalgebra of A .
4. Every near UP_i -filter of A is a near UP_s -filter of A .
5. Every UP_s -filter of A is a near UP_s -filter of A .
6. Every UP_i -filter of A is a near UP_i -filter of A .
7. Every UP_i -filter of A is a UP_s -filter of A .
8. Every UP_s -ideal of A is a UP_s -filter of A .
9. Every UP_i -ideal of A is a UP_i -filter of A .
10. Every UP_i -ideal of A is a UP_s -ideal of A .
11. Every strongly UP_s -ideal of A is a UP_s -ideal of A .
12. Every strongly UP_i -ideal of A is a UP_i -ideal of A .
13. Strongly UP_s -ideals and strongly UP_i -ideals coincide in A and it is only A .
14. The intersection of any nonempty family of fuzzy UP_s -subalgebras of A is also a fuzzy UP_s -subalgebra of A .
15. A nonempty subset S of A is a UP_s -subalgebra of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy UP_s -subalgebra of A .

16. The intersection of any nonempty family of fuzzy UP_i -subalgebras of A is also a fuzzy UP_i -subalgebra of A .
17. A nonempty subset S of A is a UP_i -subalgebra of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy UP_i -subalgebra of A .
18. The intersection of any nonempty family of fuzzy near UP_s -filters of an f - UP -semigroup $A = (A, \cdot, *, 0)$ is also a fuzzy near UP_s -filter.
19. A nonempty subset S of A is a near UP_s -filter of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy near UP_s -filter of A .
20. The intersection of any nonempty family of fuzzy near UP_i -filters of an f - UP -semigroup $A = (A, \cdot, *, 0)$ is also a fuzzy near UP_i -filter.
21. The union of any nonempty family of fuzzy near UP_i -filters of an f - UP -semigroup $A = (A, \cdot, *, 0)$ is also a fuzzy near UP_i -filter.
22. A nonempty subset S of A is a near UP_i -filter of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy near UP_i -filter of A .
23. Every fuzzy near UP_s -filter of an f - UP -semigroup is a fuzzy UP_s -subalgebra.
24. Every fuzzy near UP_i -filter of an f - UP -semigroup is a fuzzy UP_i -subalgebra.
25. The intersection of any nonempty family of fuzzy UP_s -filters of A is also a fuzzy UP_s -filter of A .
26. A nonempty subset S of A is a UP_s -filter of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy UP_s -filter of A .
27. The intersection of any nonempty family of fuzzy UP_i -filters of A is also a fuzzy UP_i -filter of A .
28. A nonempty subset S of A is a UP_i -filter of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy UP_i -filter of A .

29. Every fuzzy UP_s -filter of an f -UP-semigroup is a fuzzy near UP_s -filter.
30. Every fuzzy UP_i -filter of an f -UP-semigroup is a fuzzy near UP_i -filter.
31. The intersection of any nonempty family of fuzzy UP_s -ideals of A is also a fuzzy UP_s -ideal of A .
32. A nonempty subset S of A is a UP_s -ideal of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy UP_s -ideal of A .
33. The intersection of any nonempty family of fuzzy UP_i -ideals of A is also a fuzzy UP_i -ideal of A .
34. A nonempty subset S of A is a UP_i -ideal of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy UP_i -ideal of A .
35. Every fuzzy UP_s -ideal of A is a fuzzy UP_s -filter of A .
36. Every fuzzy UP_i -ideal of A is a fuzzy UP_i -filter of A .
37. Fuzzy strongly UP_s -ideals, fuzzy strongly UP_i -ideals, and constant fuzzy sets coincide in A .
38. The intersection and union of any nonempty family of fuzzy strongly UP_s -ideals of A are also a fuzzy strongly UP_s -ideal of A .
39. The intersection and union of any nonempty family of fuzzy strongly UP_i -ideals of A are also a fuzzy strongly UP_i -ideal of A .
40. A nonempty subset S of A is a strongly UP_s -ideal of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy strongly UP_s -ideal of A .
41. A nonempty subset S of A is a strongly UP_i -ideal of A if and only if the t -characteristic fuzzy set F_S^t is a fuzzy strongly UP_i -ideal of A .

42. Every fuzzy strongly UP_s -ideal (fuzzy strongly UP_i -ideal) of A is a fuzzy UP_s -ideal and a fuzzy UP_i -ideal of A .

43. If F is a fuzzy set in A satisfying the condition (4.3.3), then F satisfies the condition (4.3.1).

44. If F is a fuzzy set in A satisfying the condition (4.3.2), then F satisfies the condition (4.3.4).

45. If F is a fuzzy UP -subalgebra of A satisfying the condition

$$(\forall x, y \in A)(x \cdot y \neq 0 \Rightarrow f_F(x) \geq f_F(y)), \quad (4.3.5)$$

then F is a fuzzy near UP -filter of A .

46. If F is a fuzzy set in A satisfying the condition (4.3.6), then F satisfies the condition (4.3.2).

47. If F is a fuzzy near UP -filter of A satisfying the condition

$$(\forall x, y \in A)(f_F(x \cdot y) = f_F(y)), \quad (4.3.7)$$

then F is a fuzzy UP -filter of A .

48. Let A be a UP -algebra satisfying the condition

$$(\forall x, y, z \in A)(z \cdot (y \cdot x) = y \cdot (z \cdot x)). \quad (4.3.10)$$

If F is a fuzzy set in A satisfying the condition (4.3.9), then F satisfies the condition (4.3.8).

49. If F is a fuzzy set in A satisfying the condition (4.3.9), then F satisfies the condition (4.3.6).

50. If F is a fuzzy UP-filter of A satisfying the condition

$$(\forall x, y, z \in A)(f_F(y \cdot (x \cdot z)) = f_F(x \cdot (y \cdot z))), \quad (4.3.11)$$

then F is a fuzzy UP-ideal of A .

51. If F is a fuzzy set in A satisfying the condition

$$(\forall x, y, z \in A)(z \leq x \cdot y \Rightarrow f_F(z) \geq \min\{f_F(x), f_F(y)\}), \quad (4.3.13)$$

then F satisfies the condition (4.3.3).

52. If F is a fuzzy set in A satisfying the condition

$$(\forall x, y, z \in A)(z \leq x \cdot y \Rightarrow f_F(z) \geq f_F(y)), \quad (4.3.14)$$

then F satisfies the condition (4.3.3).

53. If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.3) and (3.0.14), then (\tilde{F}, E) is a fuzzy soft UP_s -subalgebra of A .

54. If (\tilde{F}, E) is a fuzzy soft UP_s -subalgebra of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft UP_s -subalgebra of A .

55. The extended intersection of two fuzzy soft UP_s -subalgebras of A is also a fuzzy soft UP_s -subalgebra. Moreover, the intersection of two fuzzy soft UP_s -subalgebras of A is also a fuzzy soft UP_s -subalgebra.

56. The union of two fuzzy soft UP_s -subalgebras of A is also a fuzzy soft UP_s -subalgebra if sets of statistics of two fuzzy soft UP_s -subalgebras are disjoint.

57. If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$

in A satisfies the conditions (4.3.3) and (3.0.15), then (\tilde{F}, E) is a fuzzy soft UP_i -subalgebra of A .

58. Every e -fuzzy soft UP_i -subalgebra of A is an e -fuzzy soft UP_s -subalgebra. Moreover, every fuzzy soft UP_i -subalgebra of A is a fuzzy soft UP_s -subalgebra.
59. If (\tilde{F}, E) is a fuzzy soft UP_i -subalgebra of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft UP_i -subalgebra of A .
60. The extended intersection of two fuzzy soft UP_i -subalgebras of A is also a fuzzy soft UP_i -subalgebra. Moreover, the intersection of two fuzzy soft UP_i -subalgebras of A is also a fuzzy soft UP_i -subalgebra.
61. The union of two fuzzy soft UP_i -subalgebras of A is also a fuzzy soft UP_i -subalgebra if sets of statistics of two fuzzy soft UP_i -subalgebras are disjoint.
62. If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.2) and (3.0.14), then (\tilde{F}, E) is a fuzzy soft near UP_s -filter of A .
63. Every e -fuzzy soft near UP_s -filter of A is an e -fuzzy soft UP_s -subalgebra. Moreover, every fuzzy soft near UP_s -filter of A is a fuzzy soft UP_s -subalgebra.
64. If (\tilde{F}, E) is a fuzzy soft UP_s -subalgebra of A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the condition (4.3.5), then (\tilde{F}, E) is a fuzzy soft near UP_s -filter of A .
65. If (\tilde{F}, E) is a fuzzy soft near UP_s -filter of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft near UP_s -filter of A .
66. The extended intersection of two fuzzy soft near UP_s -filters of A is also a fuzzy soft near UP_s -filter. Moreover, the intersection of two fuzzy soft near UP_s -filters of A is also a fuzzy soft near UP_s -filter.

67. The union of two fuzzy soft near UP_s -filters of A is also a fuzzy soft near UP_s -filter if sets of statistics of two fuzzy soft near UP_s -filters are disjoint.
68. If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.2) and (3.0.15), then (\tilde{F}, E) is a fuzzy soft near UP_i -filter of A .
69. Every e -fuzzy soft near UP_i -filter of A is an e -fuzzy soft near UP_s -filter. Moreover, every fuzzy soft near UP_i -filter of A is a fuzzy soft near UP_s -filter.
70. Every e -fuzzy soft near UP_i -filter of A is an e -fuzzy soft UP_i -subalgebra. Moreover, every fuzzy soft near UP_i -filter of A is a fuzzy soft UP_i -subalgebra.
71. If (\tilde{F}, E) is a fuzzy soft UP_i -subalgebra of A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the condition (4.3.5), then (\tilde{F}, E) is a fuzzy soft near UP_i -filter of A .
72. If (\tilde{F}, E) is a fuzzy soft near UP_i -filter of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft near UP_i -filter of A .
73. The extended intersection of two fuzzy soft near UP_i -filters of A is also a fuzzy soft near UP_i -filter. Moreover, the intersection of two fuzzy soft near UP_i -filters of A is also a fuzzy soft near UP_i -filter.
74. The union of two fuzzy soft near UP_i -filters of A is also a fuzzy soft near UP_i -filter. Moreover, the restricted union of two fuzzy soft near UP_i -filters of A is also a fuzzy soft near UP_i -filter.
75. If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.6) and (3.0.14), then (\tilde{F}, E) is a fuzzy soft UP_s -filter of A .

76. Every e -fuzzy soft UP_s -filter of A is an e -fuzzy soft near UP_s -filter. Moreover, every fuzzy soft UP_s -filter of A is a fuzzy soft near UP_s -filter.
77. If (\tilde{F}, E) is a fuzzy soft near UP_s -filter of A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the condition (4.3.7), then (\tilde{F}, E) is a fuzzy soft UP_s -filter of A .
78. If (\tilde{F}, E) is a fuzzy soft UP_s -filter of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft UP_s -filter of A .
79. The extended intersection of two fuzzy soft UP_s -filters of A is also a fuzzy soft UP_s -filter. Moreover, the intersection of two fuzzy soft UP_s -filters of A is also a fuzzy soft UP_s -filter.
80. The union of two fuzzy soft UP_s -filters of A is also a fuzzy soft UP_s -filter if sets of statistics of two fuzzy soft UP_s -filters are disjoint.
81. If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.6) and (3.0.15), then (\tilde{F}, E) is a fuzzy soft UP_i -filter of A .
82. Every e -fuzzy soft UP_i -filter of A is an e -fuzzy soft UP_s -filter. Moreover, every fuzzy soft UP_i -filter of A is a fuzzy soft UP_s -filter.
83. Every e -fuzzy soft UP_i -filter of A is an e -fuzzy soft near UP_i -filter. Moreover, every fuzzy soft UP_i -filter of A is a fuzzy soft near UP_i -filter.
84. If (\tilde{F}, E) is a fuzzy soft near UP_i -filter of A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the condition (4.3.7), then (\tilde{F}, E) is a fuzzy soft UP_i -filter of A .
85. If (\tilde{F}, E) is a fuzzy soft UP_i -filter of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft UP_i -filter of A .

86. The extended intersection of two fuzzy soft UP_i -filters of A is also a fuzzy soft UP_i -filter. Moreover, the intersection of two fuzzy soft UP_i -filters of A is also a fuzzy soft UP_i -filter.
87. The union of two fuzzy soft UP_i -filters of A is also a fuzzy soft UP_i -filter if sets of statistics of two fuzzy soft UP_i -filters are disjoint.
88. If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.8) and (3.0.14), then (\tilde{F}, E) is a fuzzy soft UP_s -ideal of A .
89. Every e -fuzzy soft UP_s -ideal of A is an e -fuzzy soft UP_s -filter. Moreover, every fuzzy soft UP_s -ideal of A is a fuzzy soft UP_s -filter.
90. If (\tilde{F}, E) is a fuzzy soft UP_s -filter of A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the condition (4.3.11), then (\tilde{F}, E) is a fuzzy soft UP_s -ideal of A .
91. If (\tilde{F}, E) is a fuzzy soft UP_s -ideal of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft UP_s -ideal of A .
92. The extended intersection of two fuzzy soft UP_s -ideals of A is also a fuzzy soft UP_s -ideal. Moreover, the intersection of two fuzzy soft UP_s -ideals of A is also a fuzzy soft UP_s -ideal.
93. The union of two fuzzy soft UP_s -ideals of A is also a fuzzy soft UP_s -ideal if sets of statistics of two fuzzy soft UP_s -ideals are disjoint.
94. If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.8) and (3.0.15), then (\tilde{F}, E) is a fuzzy soft UP_i -ideal of A .
95. Every e -fuzzy soft UP_i -ideal of A is an e -fuzzy soft UP_s -ideal. Moreover, every fuzzy soft UP_i -ideal of A is a fuzzy soft UP_s -ideal.

96. Every e -fuzzy soft UP_i -ideal of A is an e -fuzzy soft UP_i -filter. Moreover, every fuzzy soft UP_i -ideal of A is a fuzzy soft UP_i -filter.
97. If (\tilde{F}, E) is a fuzzy soft UP_i -filter of A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the condition (4.3.11), then (\tilde{F}, E) is a fuzzy soft UP_i -ideal of A .
98. If (\tilde{F}, E) is a fuzzy soft UP_i -ideal of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft UP_i -ideal of A .
99. The extended intersection of two fuzzy soft UP_i -ideals of A is also a fuzzy soft UP_i -ideal. Moreover, the intersection of two fuzzy soft UP_i -ideals of A is also a fuzzy soft UP_i -ideal.
100. The union of two fuzzy soft UP_i -ideals of A is also a fuzzy soft UP_i -ideal if sets of statistics of two fuzzy soft UP_i -ideals are disjoint.
101. Every e -fuzzy soft strongly UP_s -ideal of A is an e -fuzzy soft UP_s -ideal. Moreover, every fuzzy soft strongly UP_s -ideal of A is a fuzzy soft UP_s -ideal.
102. e -fuzzy soft strongly UP_s -ideals and e -constant fuzzy soft sets coincide in A . Moreover, fuzzy soft strongly UP_s -ideals and constant fuzzy soft sets coincide in A .
103. If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.12) (or (4.3.13) or (4.3.14)) and (3.0.14), then (\tilde{F}, E) is a fuzzy soft strongly UP_s -ideal of A .
104. If (\tilde{F}, E) is a fuzzy soft strongly UP_s -ideal of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft strongly UP_s -ideal of A .
105. The extended intersection of two fuzzy soft strongly UP_s -ideals of A is also a fuzzy soft strongly UP_s -ideal. Moreover, the intersection of two fuzzy soft strongly UP_s -ideals of A is also a fuzzy soft strongly UP_s -ideal.

106. The union of two fuzzy soft strongly UP_s -ideals is also a fuzzy soft strongly UP_s -ideal. Moreover, the restricted union of two fuzzy soft strongly UP_s -ideals of A is also a fuzzy soft strongly UP_s -ideal.
107. Every e -fuzzy soft strongly UP_i -ideal of A is an e -fuzzy soft UP_i -ideal. Moreover, every fuzzy soft strongly UP_i -ideal of A is a fuzzy soft UP_i -ideal.
108. e -fuzzy soft strongly UP_i -ideals and e -constant fuzzy soft sets coincide in A . Moreover, fuzzy soft strongly UP_i -ideals and constant fuzzy soft sets coincide in A .
109. If (\tilde{F}, E) is a fuzzy soft set over A such that for all $e \in E$, a fuzzy set $\tilde{F}[e]$ in A satisfies the conditions (4.3.12) (or (4.3.13) or (4.3.14)) and (3.0.15), then (\tilde{F}, E) is a fuzzy soft strongly UP_i -ideal of A .
110. If (\tilde{F}, E) is a fuzzy soft strongly UP_i -ideal of A and $\emptyset \neq E^* \subseteq E$, then $(\tilde{F}|_{E^*}, E^*)$ is a fuzzy soft strongly UP_i -ideal of A .
111. The extended intersection of two fuzzy soft strongly UP_i -ideals of A is also a fuzzy soft strongly UP_i -ideal. Moreover, the intersection of two fuzzy soft strongly UP_i -ideals of A is also a fuzzy soft strongly UP_i -ideal.
112. The union of two fuzzy soft strongly UP_i -ideals of A is also a fuzzy soft strongly UP_s -ideal. Moreover, the restricted union of two fuzzy soft strongly UP_i -ideals of A is also a fuzzy soft strongly UP_i -ideal.
113. The restricted union of family of fuzzy soft near UP_i -filters of A is also a fuzzy soft near UP_i -filter.
114. The union of family of fuzzy soft near UP_i -filters of A is also a fuzzy soft near UP_i -filter.
115. The intersection of family of fuzzy soft UP_s -subalgebras of A is also a fuzzy soft UP_s -subalgebra.

116. The extended intersection of family of fuzzy soft UP_s -subalgebras of A is also a fuzzy soft UP_s -subalgebra.

117. The AND of family of fuzzy soft UP_s -subalgebras of A is also a fuzzy soft UP_s -subalgebra.

118. The OR of family of fuzzy soft near UP_i -filters of A is also a fuzzy soft near UP_i -filter.

119. Let (\tilde{F}_i, E_i) and (\tilde{F}, E) be fuzzy soft sets over a common universe U where I is a nonempty set. Then the following properties hold:

$$(1) (\tilde{F}, E) \cap (\bigcup_{i \in I} (\tilde{F}_i, E_i)) = \bigcup_{i \in I} ((\tilde{F}, E) \cap (\tilde{F}_i, E_i)),$$

$$(2) (\bigcup_{i \in I} (\tilde{F}_i, E_i)) \cap (\tilde{F}, E) = \bigcup_{i \in I} ((\tilde{F}_i, E_i) \cap (\tilde{F}, E)),$$

$$(3) (\tilde{F}, E) \cup (\bigcap_{i \in I} (\tilde{F}_i, E_i)) = \bigcap_{i \in I} ((\tilde{F}, E) \cup (\tilde{F}_i, E_i)),$$

$$(4) (\bigcap_{i \in I} (\tilde{F}_i, E_i)) \cup (\tilde{F}, E) = (\tilde{F}_i, E_i) \cup \bigcap_{i \in I} ((\tilde{F}, E)),$$

$$(5) (\tilde{F}, E) \cap (\bigcup_{i \in I} (\tilde{F}_i, E_i)) = \bigcup_{i \in I} ((\tilde{F}, E) \cap (\tilde{F}_i, E_i)),$$

$$(6) (\bigcup_{i \in I} (\tilde{F}_i, E_i)) \cap (\tilde{F}, E) = \bigcup_{i \in I} ((\tilde{F}_i, E_i) \cap (\tilde{F}, E)),$$

$$(7) (\tilde{F}, E) \cup (\bigcap_{i \in I} (\tilde{F}_i, E_i)) = \bigcap_{i \in I} ((\tilde{F}, E) \cup (\tilde{F}_i, E_i)),$$

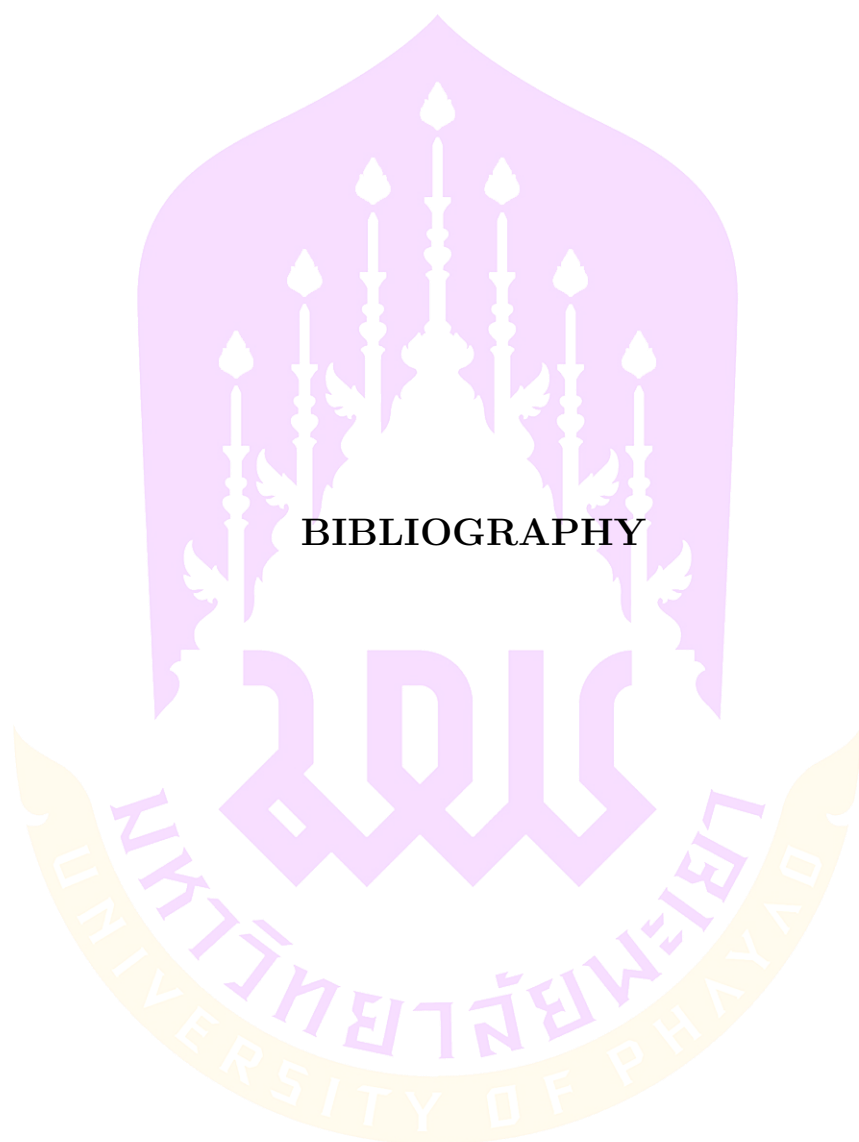
$$(8) (\bigcap_{i \in I} (\tilde{F}_i, E_i)) \cup (\tilde{F}, E) = \bigcap_{i \in I} ((\tilde{F}_i, E_i) \cup (\tilde{F}, E)),$$

$$(9) (\tilde{F}, E) \cap (\bigcup_{i \in I} (\tilde{F}_i, E_i)) = \bigcup_{i \in I} ((\tilde{F}, E) \cap (\tilde{F}_i, E_i)),$$

$$(10) (\bigcup_{i \in I} (\tilde{F}_i, E_i)) \cap (\tilde{F}, E) = \bigcup_{i \in I} ((\tilde{F}_i, E_i) \cap (\tilde{F}, E)),$$

$$(11) (\tilde{F}, E) \cup (\bigcap_{i \in I} (\tilde{F}_i, E_i)) = \bigcap_{i \in I} ((\tilde{F}, E) \cup (\tilde{F}_i, E_i)), \text{ and}$$

$$(12) (\bigcap_{i \in I} (\tilde{F}_i, E_i)) \cup (\tilde{F}, E) = \bigcap_{i \in I} ((\tilde{F}_i, E_i) \cup (\tilde{F}, E)).$$



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